



REPUBLIC OF TRINIDAD AND TOBAGO
MINISTRY OF EDUCATION

Secondary Education Modernization Programme

SECONDARY SCHOOL CURRICULUM

Forms 4 - 5

Mathematics

Curriculum Planning and Development Division, Ministry of Education
2009

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Minister's Foreword

The Government of The Republic of Trinidad and Tobago, in its *Vision 2020 Draft National Strategic Plan*, has articulated a vision of “a united, resilient, productive, innovative and prosperous nation with a disciplined, caring, fun-loving society comprising healthy, happy and well-educated people and built on the enduring attributes of self-reliance, respect, tolerance, equity and integrity.”

In order to achieve this vision, the nation must ensure that its learners receive a high quality education. This requires that the Ministry of Education make quality education a national priority and that schools make it an institutional imperative.

Curriculum Guides are central to guiding the process to achieving quality. They set the standards for all stakeholders who have an input in the final years of secondary level education. They align the three critical elements of our education system: the teaching/learning process, resources, and the contribution of stakeholders.

We expect that teachers will implement these Guides to ensure that their lessons are relevant to the expressed needs of The Republic of Trinidad and Tobago, while simultaneously meeting the varied needs of the students. It is intended that this outcome will be achieved in a climate where students are taught in ways that are appropriate to their individual learning styles. The Guides support a teaching/learning process that is based on a curriculum that is itself rooted in sound and well established educational theories and practice.

On behalf of the Ministry of Education I thank all those who contributed to the development of these Curriculum Guides



Hon. Esther Le Gendre
Minister of Education
The Republic of Trinidad and Tobago

A Note to Teachers

These Curriculum Guides have been developed by educators, including practising teachers, for teachers. They are intended to assist you to prepare students to meet the rapidly changing demands of life in the 21st century, while ensuring that they acquire the core of general knowledge and experience essential for later education and employment. The new curriculum that they represent is designed to guide the adoption of a more student-centred approach to instruction and the provision of learning opportunities that are relevant to today's students and inclusive of varied learning needs and interests.

Since the beginning of the curriculum development process, we have seen profound changes in the use of technology in education and there is no doubt that similar shifts will take place in the coming years. The challenge for us as educators is to find ways to make our approach to teaching flexible, progressive, and responsive, so that we embrace and motivate change where it benefits learners. This entails becoming lifelong learners ourselves and creating environments that provide necessary community support and foster professional development.

The Guides embody the culmination of seven years of development and revision activity. The National Curriculum will, however, be regularly reviewed to ensure that it continues to meet the needs of all students and matches the goals of society. Your input in this process is vital and we welcome and encourage your ongoing feedback.

Instructional decisions must be based on sound, contemporary educational theory, practice, and research. These documents will serve as important guides for the development of instructional programmes to be implemented at the school and classroom levels. They are organized in several parts. Part 1 is common to all and provides the general philosophy and aims in which every subject is anchored. Part 2 is specific to each subject and includes specific outcomes and sample activities and strategies that may be used to achieve them. The rest of the document is designed to suit the particular needs of each subject area. All the Guides include suggested assessment strategies and recommended resources.

We in the Curriculum Planning and Development Division are confident that the new National Curriculum Guides for Forms 4 and 5 will contribute significantly to enhanced teaching and learning experiences in our secondary schools and, consequently, the achievement of personal learning and national educational goals.

Sharon Douglass-Mangroo
Director of Curriculum Development
August 2008

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The Ministry of Education wishes to express its sincere appreciation to all those who contributed to the curriculum development and revision processes from 2000 to the present.

- The staff of the Coordinating Unit of the Secondary Education Modernization Programme (SEMPCU), past and present, provided technical assistance and planned, organized, and conducted the various exercises over the years. They include Mr. Maurice Chin Aleong, Mr. Lloyd Pujadas, Ms. Patricia Sealey, Mr. Arnott West, Dr. Stephen Joseph, Ms. Renee Figuera, and Ms. Roslyn Elias.
- Mrs. Sharon Douglass-Mangroo, Director of Curriculum Development, led the curriculum development sub-component and coordinated the curriculum development and revision activities.
- Dr. Robert Sargent guided the early curriculum development process.
- The Principals of the pilot schools generously contributed teachers and participated in regular meetings to provide valuable feedback on field tests.
- The Principals of non-pilot schools kindly released teachers to take part in writing activities.
- The staff of the School Libraries Division actively joined in workshops, facilitated research, and contributed to the infusion of information technology into the curriculum.
- Editors, past and present: Ms. Avril Ross, Ms. Lynda Quamina-Aiyejina, and Ms. Patricia Worrell devoted time, energy, and knowledge to editing the several versions of the documents.
- The Administrative staff of the Curriculum Development Division spent long hours typing and retyping the documents.
- Officers of the Divisions of Educational Services, Schools Supervision, Student Support Services, and Educational Research and Evaluation provided support as needed.
- Teachers throughout the secondary school system responded to requests for comments and other forms of feedback.
- The Curriculum Officers and members of the Curriculum Writing Teams brought their knowledge, skills and practical experiences of teaching and learning to the curriculum development workshops and skillfully synthesized all to produce these documents.

1



THE NATIONAL CURRICULUM FOR FORMS 4 AND 5



Background

From the Ministry of Education's *Corporate Plan 2008–2012* (p. 4)

The Government of Trinidad and Tobago, in its *Vision 2020 Draft National Strategic Plan*, has articulated a vision of “a united, resilient, productive, innovative, and prosperous nation with a disciplined, caring, fun-loving society comprising healthy, happy and well-educated people and built on the enduring attributes of self reliance, respect, tolerance, equity and integrity...”

Towards the achievement of this Vision, the Government has articulated five developmental pillars:

Developing Innovative People

Nurturing a Caring Society

Governing Effectively

Enabling Competitive Business

Investing in Sound Infrastructure and Environment

The Ministry of Education has been identified as one of the champions for *developing innovative people*. Central to the realization of this pillar is “A highly skilled, well-educated people aspiring to a local culture of excellence that is driven by equal access to learning opportunities.”

In conjunction with other key Ministries, the Ministry of Education has been charged with the realization of the following goals:

The people of Trinidad and Tobago will be well known for excellence in innovation.

Trinidad and Tobago will have a seamless, self-renewing, high-quality education system.

A highly skilled, talented and knowledgeable workforce will stimulate innovation driven growth and development.

The richness of our diverse culture will serve as a powerful engine to inspire innovation and creativity.

Nationally, the reform of the education system is driven by several local, regional and international perspectives. We are committed to a seamless, self-renewing, high-quality education system underpinned by a National Model for Education. This National Model has three (3) foci as follows:

- I. To ensure an alignment of the Education System to Government's Strategic Plan Vision 2020 which mandates that the education system produces caring and innovative citizens
- II. To ensure that the Education System produces citizens with a sense of democracy, respect for the rights of others and elders and with the ability to contribute meaningfully to the social and economic development of the country
- III. To build a strong sense of nationalism and patriotism in our citizens. (p. 7)

The Secondary Curriculum

In its commitment to comprehensive reform and expansion of the secondary school system, the Government of the Republic of Trinidad and Tobago, in 1996, adopted the report of the National Task Force on Education as educational policy. The specific recommendations for the improvement of secondary education led to discussions with the Inter-American Development Bank (IDB) for loan funding arrangements for a programme, the Secondary Education Modernization Programme (SEMP), to modernize secondary education in Trinidad and Tobago. One of the intended outcomes of this programme was improved educational equity and quality.

The Curriculum Guides produced for Forms 4 & 5 in eight subject areas are among the products and contribute to this outcome.

The Curriculum Underpinnings

The new curriculum has been informed by a wealth of available curriculum theories and processes.

The major forces that influence and shape the organization and content of the curriculum originate from:

1. Educational philosophy and the nature of knowledge
2. Society and culture
3. The learner and learning process
4. The nature and structure of subject matter to be learned
5. Learning theories

Considerations of these areas represent the foundation on which the National Curriculum is built. The philosophical concerns and educational goals that shaped the curriculum also formed the basis for the dialogue with stakeholders in which the Curriculum Development Division engaged, with the aim of developing a coherent, culturally focussed, and dynamically evolving curriculum.

An internal analysis of the Education System, together with the research conducted in international forums, has shown that the curriculum is core to the development of innovative people. This curriculum is aimed at attaining six essential learning outcomes. The six outcomes identified help to define universally accepted goals that have been developed and underscored by other educational jurisdictions and that have been agreed to be essential. The essential learning outcomes help to define standards of attainment for all secondary school students.

The Essential Learning Outcomes

The learning outcomes deemed essential are in the areas of:

- Aesthetic Expression
- Citizenship
- Communication
- Personal Development
- Problem Solving
- Technological Competence

The achievement of these Essential Learning Outcomes by all students is the goal that every core curriculum subject must facilitate. The core curriculum subjects; their content; and the teaching, learning, and assessment strategies are the means to fulfill this end.

It is expected that by the end of the third year of secondary school, students' achievement in all six areas will result in a solid foundation of knowledge, skills, and attitudes that will constitute a platform for living in the Trinidad and Tobago society and making informed choices for further secondary education.

The Essential Learning Outcomes are described more fully below.

Aesthetic Expression

Students recognize that the arts represent an important facet of their development, and they should respond positively to its various forms. They demonstrate visual acuity and aesthetic sensibilities and sensitivities in expressing themselves through the arts.

Students, for example:

- use various art forms as a means of formulating and expressing ideas, perceptions, and feelings;
- demonstrate understanding of the contribution of the arts to daily life, cultural identity, and diversity;
- demonstrate understanding of the economic role of the arts in the global village society;
- demonstrate understanding of the ideas, perceptions, and feelings of others as expressed in various art forms;
- demonstrate understanding of the significance of cultural resources, such as museums, theatres, galleries, and other expressions of the multicultural reality of society.

Citizenship

Students situate themselves in a multicultural, multi-ethnic environment, and understand clearly the contribution they must make to social, cultural, economic, and environmental development in the local and global context.

Students, for example:

- demonstrate understanding of sustainable development and its implications for the environment locally and globally;
- demonstrate understanding of Trinidad and Tobago's political, social, and economic systems in the global context;
- demonstrate understanding of the social, political, and economic forces that have shaped the past and present, and apply those understandings to the process of planning for the future;
- examine issues of human rights and recognize and react against forms of discrimination, violence, and anti-social behaviours;

- determine the principles and actions that characterize a just, peaceful, pluralistic, and democratic society, and act accordingly;
- demonstrate understanding of their own cultural heritage and cultural identity, and that of others, as well as the contribution of the many peoples and cultures to society.

Communication

Students use their bodies, the symbols of the culture, language, tools, and various other media to demonstrate their deeper understandings of synergies inherent in the exchange of ideas and information, and thus to communicate more effectively.

Students, for example:

- explore, reflect on, and express their own ideas, learning, perceptions, and feelings;
- demonstrate understanding of facts and relationships presented through words, numbers, symbols, graphs, and charts;
- demonstrate sensitivity and empathy where necessary in communicating various kinds of emotions and information
- present information and instructions clearly, logically, concisely, and accurately for a variety of audiences;
- interpret and evaluate data, and express their conclusions in everyday language;
- critically reflect on and interpret ideas presented through a variety of media.

Personal Development

Students “grow from inside out,” continually enlarging their knowledge base, expanding their horizons, and challenging themselves in the pursuit of a healthy and productive life.

Students, for example:

- demonstrate preparedness for the transition to work and further learning;
- make appropriate decisions and take responsibility for those decisions;
- work and study purposefully, both independently and in cooperative groups;
- demonstrate an understanding of the relationship between health and lifestyle;
- discriminate among a wide variety of career opportunities;
- demonstrate coping, management, and interpersonal skills;
- display intellectual curiosity, an entrepreneurial spirit, and initiative;
- reflect critically on ethical and other issues;
- deal effectively with change and become agents for positive, effective change.

Problem Solving

Students know problem-solving strategies and apply them to situations they encounter. They demonstrate critical thinking and inquiry skills with which they process information to solve a wide variety of problems.

Students, for example:

- acquire, process, and interpret information critically to make informed decisions;
- use a variety of strategies and perspectives flexibly and creatively to solve problems;
- formulate tentative ideas, and question their own assumptions and those of others;
- solve problems individually and collaboratively;
- identify, describe, formulate, and reformulate problems;
- frame and test hypotheses;
- ask questions, observe relationships, make inferences, and draw conclusions;
- identify, describe, and interpret different points of view;
- distinguish facts from opinions.

Technological Competence

Students are technologically literate, understand and use various technologies, and demonstrate an understanding of the role of technology in their lives, in society, and in the world at large.

Students, for example:

- locate, evaluate, adapt, create, and share information using a variety of sources and technologies;
- demonstrate understanding of and use existing and developing technologies appropriately;
- demonstrate understanding of the impact of technology on society;
- demonstrate understanding of ethical issues related to the use of technology in local and global contexts.

The Curriculum Design and Development Process

In order to achieve the outcomes defined by the underpinning philosophy and goals, the Curriculum Development Division of the Ministry of Education embarked on a design and development programme consonant with accepted approaches to curriculum change and innovation.

Curriculum Design

This curriculum displays a learner-centered design. Its philosophical assumptions are mainly constructivist. Its major orientation is to curriculum as self-actualization. The curriculum is student-centred and growth oriented. It seeks to provide personally satisfying experiences for each student. As the student moves from one level to another, activities also expand to allow him/her new insights and approaches to dealing with and integrating new knowledge.

Curriculum Development

The first stage of the curriculum development process consisted of consultations with stakeholders from a cross-section of the national community. Consultations were held with primary and secondary school teachers; principals; members of denominational school boards; members of the business community; the executive of the Trinidad and Tobago Unified Teachers' Association (TTUTA); representatives from The University of the West Indies (UWI), John S. Donaldson Technical Institute, San Fernando Technical Institute, Valsayn Teachers' College and Caribbean Union College; parents; librarians; guidance counsellors; students; curriculum officers; and school supervisors. They were focused on the philosophy, goals, and learning outcomes of education.

The result of these consultations was agreement on:

- the concept of a “core,” that is, essential learning outcomes consisting of skills, knowledge, attitudes, and values that students must acquire at the end of five years of secondary schooling;
- the eight subjects to form the core;
- the desirable outcomes of secondary school education in Trinidad and Tobago.

In Stage 2 of the process, the officers of the Curriculum Development Division studied the reports of the consultations, the Education Policy Paper, the reports of the Curriculum Task Force and the Task Force for Removal of Common Entrance, as well as newspaper articles and letters to the editor on education during the preceding five years. The School Libraries Division and the Division of School Supervision assisted the Curriculum Development Division in this task. The result of the study was the identification and articulation of a set of desirable outcomes and essential exit competencies to be possessed by all students on leaving school. All learning opportunities, all teaching and learning strategies, and all instructional plans are to contribute to the realization of these outcomes and competencies.

At Stage 3, 10 existing schools were identified to pilot the new curriculum. Teachers from eight subject areas were drawn from these schools to form Curriculum Writing Teams for each subject. Teachers with specific subject or curriculum development skills from other schools were also included in the teams. The outputs of this phase included learning outcomes specific to each subject that contribute to the fulfillment of the national outcomes; subject content; and teaching, learning, and assessment strategies to support the outcomes.

The draft Curriculum Guides for Forms 1 and 2 were approved by Cabinet for introduction into schools on a phased basis in September 2003. The draft guides for Form 3 were completed and introduced in the following year. Introduction of the new guides was accompanied by professional development and training for principals and teachers. The Ministry also began to supply new and/or upgraded facilities for teaching and learning, and educational technology. At the same time, work began on a new assessment and certification system.

Curriculum Revision

As implementation proceeded, feedback was received in the Curriculum Development Division through school visits, workshops, and reviews by UWI lecturers and other stakeholders. In 2007, a survey was conducted among teachers, followed by focus group meetings, in order to concretize feedback before embarking on the revision process. As in the original curriculum development exercise, revision—the final stage—was carried out by teams of practising teachers led by officers of the Curriculum Development Division.

Teaching of English Language across the Curriculum

Language is a uniquely human capacity. The development of language skills and the ability to understand and use language correctly, competently, and effectively is fundamental to the learning outcomes expressed in the national curriculum. Three simultaneous kinds of learning are envisaged: students learn language, they learn through language, and they learn about language.

The National Curriculum envisages that language development of students takes place across the curriculum and is therefore to be addressed in all subject areas. Students will develop and use patterns of language vital to understanding and expression in the different subjects that make up the curriculum.

Language plays a major role in learning and occurs when students use the major modes of language—listening, speaking, reading, and writing—to achieve various purposes, among them: to communicate with others; to express personal beliefs, feelings, ideas, and so on; for cognitive development in various subjects of the curriculum; and to explore and gain insight into and understanding of literature. Language is linked to the thinking process, and its use allows students to reflect on and clarify their own thought processes and, thus, their own learning.

The student of Trinidad and Tobago functions in a bidialectal context, that is, the natural language of the student, the Creole, differs from the target language and the language of instruction, Internationally Acceptable English. Both languages are of equal value and worth and are to be respected. Students use their own language as a tool for interpreting the content of the curriculum and for mastering it, and are to be taught to use the target language as effectively and effortlessly as they would their natural language.

The exponential growth in information and the use of information and communication technologies provide the opportunity for students to be critical users of information. Language development and use in this context is also addressed in all subject areas.

Education Policies that Impact on the Curriculum

There are several Ministry of Education policies that impact on the National Secondary Curriculum, though some are still in the process of formalization. These include the National Model for Primary and Secondary Education in Trinidad and Tobago, the ICT policy, Standards for the Operation of Schools, and Quality Standards. Copies of these documents may be obtained from the Ministry offices or the website at www.moe.gov.tt. Three policies that have direct impact on the development and implementation of the curriculum are discussed below.

National Curriculum Policy

A Draft National Curriculum Policy has been approved by Cabinet for consultation with stakeholders. The Policy statements are summarized as follows:

1. The curriculum must articulate with the goals of national development and be supportive of the aspirations of individuals and their personal development. It must provide opportunities for every student to be equipped with the knowledge, skills, attitudes, values, and dispositions necessary for functioning in an interactive, interdependent society.
2. The curriculum must be so managed as to ensure the provision of a quality curriculum experience for all students at all levels of the system.
3. At every level of the system, there must be equitable provision of requisite facilities, resources, services, and organizational structures that are conducive to and supportive of effective learning and teaching and healthy development.
4. Continuous quality management must support all curriculum and related activities at every level of the system.
5. Ongoing research and professional development activities must equip education practitioners for continued effective practice.

Though not yet formally accepted, these statements are worthy of consideration at all stages of the curriculum cycle.

Inclusive Education Policy

The Ministry of Education is committed to “support the delivery of inclusive education in all schools by providing support and services to all learners, and by taking appropriate steps to make education available, accessible, acceptable and adaptable to all learners.” An inclusive curriculum is acknowledged to be the most important factor in achieving inclusive education. In planning and teaching the school curriculum, teachers are therefore required to give due regard to the following principles:

- The National Curriculum Guides set out what most students should be taught at lower secondary school but teachers should teach the required knowledge and skills in ways that suit students’ interests and abilities. This means exercising flexibility and drawing from curricula for earlier or later class levels to provide learning opportunities that allow students to make progress and experience success. The degrees of differentiation exercised will depend on the levels of student attainment.
- Varied approaches to teaching, learning, and assessment should be planned to allow all students to participate fully and effectively. Account should be taken of diverse cultures, beliefs, strengths, and interests that exist in any classroom and that influence the way students learn.
- Students with special needs shall receive additional instructional support in the context of the regular curriculum, not a different one. The guiding principle of equity is to supply students who need it with additional help to achieve set standards rather than to lower the standards.
- Continuous formative evaluation must be used to identify learning needs and to shape instruction, thus maximizing students’ opportunities for achieving success. Assessment strategies must be appropriate to the way the curriculum is designed and delivered, as well as to each student’s individual learning profile and stage of development.
- Suitable technology shall be used in instruction to facilitate learning and enhance success.

ICT in the Curriculum

The following statements are taken from the Ministry of Education's ICT in Education Policy (pp. 28–29).

Curriculum Content and Learning Resources

- Curriculum and content must increasingly maximize the use of ICT.
- ICT must be integrated into the development and delivery of the curriculum.
- ICT integration and ICT competency measures across the curriculum shall be driven through the development and delivery of an ICT-infused curriculum.

The Core Curriculum Subjects

These are subjects for which every student is required to demonstrate achievement of the stated outcomes in Forms 4 and 5. Additional subjects that contribute to students' holistic development and further their interests and aspirations may also be offered thereafter.

A minimum time allocation is recommended for each core subject. The Principal, as instructional leader of the school, will make the final decision according to the needs of the students and the resources available at any given time.

The subjects and the time allocations are as follows:

Subject	No. of Periods	Subject	No. of Periods
English/Language Arts	6	Mathematics	5
Science	4	Health and Physical Education	2
Spanish	4	Visual and Performing Arts	4
Social Studies	4		

At the end of Form 5, students will be assessed for the National Certificate of Secondary Education (NCSE), Level 2.

Curriculum Implementation

Implementation of the curriculum is a dynamic process, requiring collaboration of the developers (curriculum teams) and users (teachers). In implementation, teachers are expected to use the formal curriculum, as described in the Curriculum Guides, to plan work and teach in a manner that accomplishes the objectives described. Teachers translate those objectives into units of study, determining the appropriate sequence and time allocation according to the learning needs of their students. The new Curriculum Guides provide sample teaching and assessment strategies but it is the role of the professional teacher to select and use sound teaching practices, continually assessing student learning, and systematically providing feedback to the curriculum team for use in revising and improving the guides.

The Curriculum Development System advocated by the Ministry of Education involves stakeholders, specialist curriculum officers, principals, heads of departments, and teachers, each with specific roles and responsibilities. Some of these are outlined in the table below.

SYSTEM COMPONENT	MEMBERS	ROLE
National Curriculum Council	Stakeholders	<ul style="list-style-type: none"> • Advise on curriculum policy, goals, and standards
Curriculum Planning and Development Division (Head Office and District based)	Curriculum Officers	<ul style="list-style-type: none"> • Curriculum planning • Provide leadership in identifying curriculum goals and determining the process for development of curriculum materials • Lead writing teams (includes teachers) • Monitor implementation • Provide teacher support • Advise on processes and materials for effective implementation and student assessment • Evaluate curriculum
School Curriculum Council	Principal/Vice Principal and Heads of Departments	<ul style="list-style-type: none"> • Make major decisions concerning the school curriculum such as assigning resources • Provide guidelines for Instructional Planning Teams
Instructional Planning Teams/School Instructional Committees	Teachers	<ul style="list-style-type: none"> • Cooperate on tasks necessary for effective implementation, such as: yearly work plans, units of study, development of materials to individualize the curriculum, identification and development of learning materials, student assessment and evaluation

Curriculum Implementation at School Level

The “School Curriculum” refers to all the learning and other experiences that the school plans for its students. It includes the formal or written curriculum as well as all other learning activities, such as those offered by student clubs, societies, and committees, as well as sporting organizations (e.g., cricket team, debating society, Guides, Cadets).

The School Curriculum Council develops the School Curriculum in alignment with the National Curriculum. It consists of the Principal and/or Vice Principal and Heads of Department. The duties of the Council include the development of school culture, goals, vision, and curriculum in alignment with the national curriculum and culture. It also provides support for curriculum work and performs evaluation functions.

In providing support for curriculum work, the Council:

- encourages teachers to identify challenges and try new ideas;
- timetables to allow for development of curriculum materials, for example, year plans, units, instructional materials;
- ensures availability of learning materials;
- provides instructional leadership;
- ensures appropriate strategies for student success.

In performing evaluation functions, the Council:

- monitors the curriculum (observation, test scores, student books, talks);
- assesses the hidden curriculum (discipline policies, fund allocation, physical environment);
- evaluates the school programme of studies.

The roles of the instructional teams and the individual teacher are described in the following tables:

Role of School Instructional Committees
Develop/Revise/Evaluate work programmes
Determine resource needs
Identify/Develop instructional materials
Conduct classroom action research
Integrate and align curriculum
Identify and develop appropriate assessment practices
Develop reporting instruments and procedures (student and teacher performance)
Keep records

Role of the Individual Teacher
Develop/Revise instructional programme
Individualize curriculum to suit students needs and interests
Develop/Evaluate/Revise unit plans
Develop/Select appropriate learning materials
Select appropriate teaching strategies to facilitate student success
Integrate as far as possible and where appropriate
Select appropriate assessment strategies
Monitor/Assess student learning; Keep records
Evaluate student performance
Evaluate classroom programmes
Conduct action research
Collaborate with colleagues

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MATHEMATICS

2

FORMS 4 AND 5

CURRICULUM CONTENT SUBJECT MODULES



Acknowledgements
Members of the Original Curriculum Team

NAME	SCHOOL /INSTITUTION
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11. Don Naranjit	Curriculum Officer
12. John Roopchan	Curriculum Officer

THE MATHEMATICS CURRICULUM

The background of the slide is a blackboard filled with various mathematical notations and diagrams. At the top, the quadratic formula is written: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. To the left, there's a diagram of a right-angled triangle with angles α and β , and sides labeled x , y , and z . Below this, the sine rule is written: $\frac{\sin \alpha}{\sin a} = \frac{24x4}{3z}$. To the right, the word "quadratic" is written next to the formula. Further right, the word "Equation" is written next to $\sqrt{49} = 7$. At the bottom, the word "Pythagoras" is written next to the equation $2y - 3y = x$. A small table is also visible on the right side of the slide.

x	y
6	7
7	2
9	10
3	3
9	7
12	11

VISION

A nation driven by Mathematics and recognized on the world stage

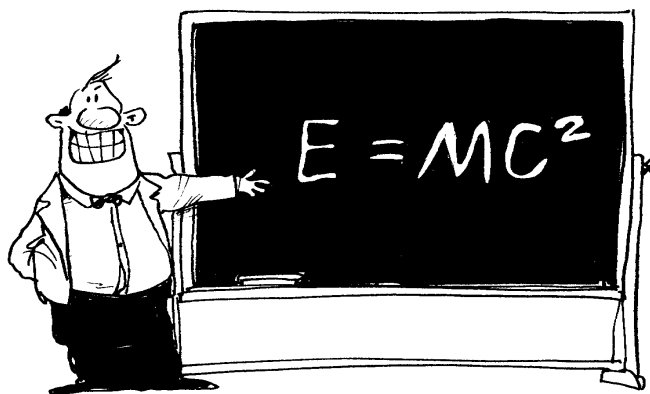
MISSION

To foster the growth and development of mathematically empowered students in Trinidad and Tobago so that they can effectively contribute to our society and serve as catalysts to world development

RATIONALE FOR TEACHING AND LEARNING MATHEMATICS

Reports on mathematical achievement from external examination bodies, the Ministry of Education, employers, and public and private agencies have all concluded that the majority of students at both primary and secondary levels lack basic skills in numeracy. The high percentage of students who are not presently certified as being proficient in Mathematics is one indicator of the problem. Many factors must be considered in addressing this situation but the most important must be the design of a Mathematics Curriculum that is relevant to the needs of learners and of society.

Mathematics is an activity that is critical for the development of individuals and societies. It is the study of the properties of number and its relationship to measurement, space, shape, statistics, and probability. Mathematics also deals with abstractions, with algebra being the strand of mathematics that presents abstraction in its purest form. The study of Mathematics enables individuals to become creative and critical thinkers through the development of logical thinking, problem-solving, investigative, organizational, and argumentative skills.



Students acquire Mathematical power by constructing Mathematical knowledge and understanding. Mathematically empowered students can adapt to the quickening pace of change in today's society. They will have acquired basic skills, self-confidence, and self-reliance, which will prepare them to make effective contributions to their society. Through experiencing and practising the processes of communication, reasoning, making connections and representations and recognizing patterns and relationships, students will have attained the essential learning outcomes identified for the National Curriculum and, inevitably, the goals of education.

Mathematics is also essential to the study of all other subjects on both the primary and secondary schools' curriculum and this in itself underscores its value and the role it plays in our lives.

The philosophy of education that informs this Mathematics Curriculum is underpinned by a belief that all children can learn, but that children learn in diverse ways. In order to ensure that all students become mathematically proficient, the Curriculum is informed by current research on the nature and purpose of Mathematics as well as on the pedagogy.

Since Mathematics pervades our daily lives, the Mathematics Curriculum also reflects the various ways in which students encounter Mathematics in their environment and in real-life situations. Thus, the new Curriculum represents a major shift in the paradigm of understanding Mathematics and how it is taught, learnt and used.

For the most part, Mathematics is conceptualized as a process that can be carried out in different ways and using multiple approaches to achieve similar ends. The curriculum focus is on doing Mathematics, as far as possible, in real-life contexts. To this end, the Curriculum advocates the use of performance tasks in the learning process, so as to make classroom experiences more authentic. Assessment practices should also be aligned to meaningful tasks.

THE PURPOSE AND STRUCTURE OF THE CURRICULUM

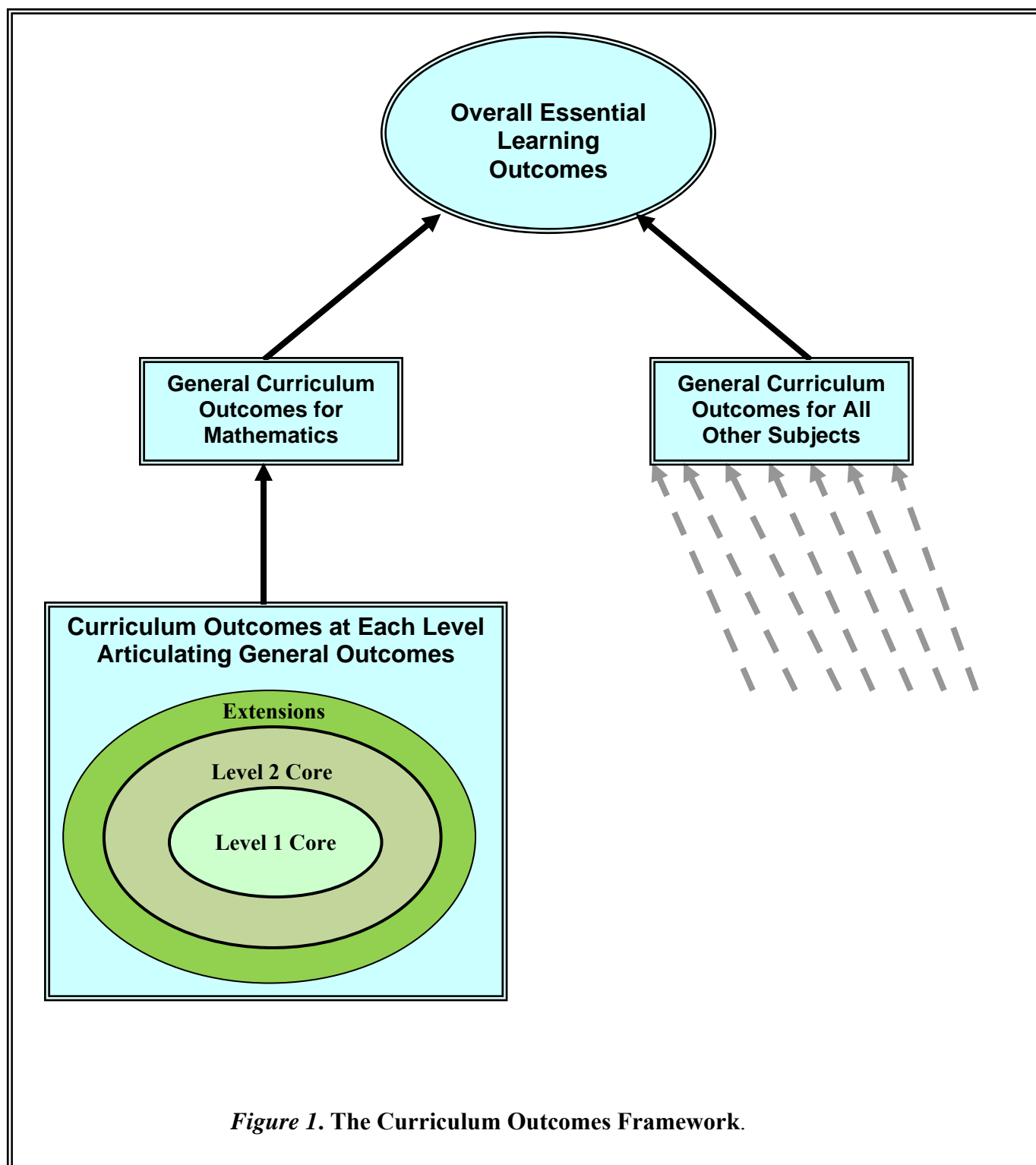
Traditionally, Mathematics was accepted as the domain of an elite few who were able to master complex skills and understand abstract concepts. For many, learning Mathematics was conceptualized narrowly as recalling facts and procedures that, in many instances, had no relevance to the real world and served little purpose beyond examinations and certification.

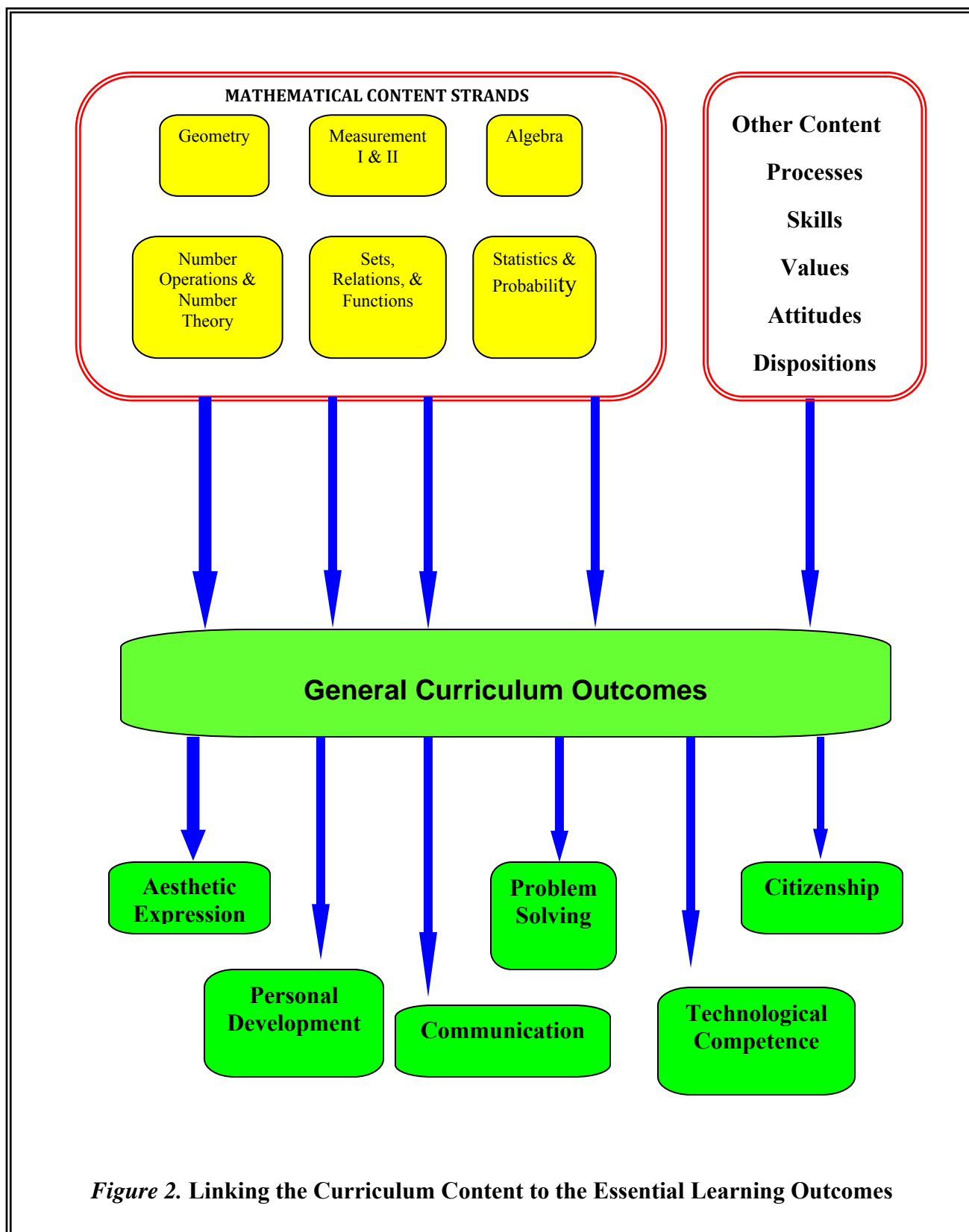
The approach to learning Mathematics taken in this Curriculum should bring many benefits to the individual and to the society as a whole. Among these benefits is the basic support that Mathematics lends to the other core curriculum areas in the attainment of the six Essential Learning Outcomes, which are seen as the hallmark characteristics of all graduates. These educational Learning Outcomes provide the critical impetus for the thrust of the new Curriculum.

The Curriculum is intended to take learners through a spiralling learning process that integrates learning through content and pedagogy. A modular approach has been taken in designing the Curriculum. Within this modular design, appropriate themes capture cross-sections of the traditional Curriculum content, which is used primarily as a vehicle for developing the necessary competencies and understandings that will help students to attain the six Essential Learning Outcomes.

The five modules in the curriculum are: **Exploring Numbers; Using the Evidence; Modelling with Mathematics; Reasoning with Space; and Taking Control of Space.**

The content of the Form 4 and 5 Curriculum includes both core content and extensions. Every student in Trinidad and Tobago is expected to master at least the material in the core. A student who has mastered the material in both the core and extensions, should be able to perform well in Mathematics at secondary exit examinations such as the Caribbean Secondary Education Certificate (CSEC) Examination and other comparable examinations offered by other examining bodies. Moreover, students who master the skills, acquire the knowledge and understanding of the content, and develop the attitudes, values and dispositions as suggested in the core, should be well positioned to attain the overall outcomes targeted in the Curriculum Guides.





MATHEMATICS CURRICULUM

CONTENT AND PROCESS OUTCOMES AS OUTLINED BY NCTM

The National Council of Teachers of Mathematics (NCTM) of the United States of America has organized Mathematics Outcomes in content and process strands. Some of these are given below.

Number and Operations

1. Understand number representation
2. Understand number relationships and number systems
3. Understand operations, their relationships, and how they relate to number
4. Compute fluently
5. Make reasonable estimates

Algebra

1. Understand patterns, relations, and functions
2. Represent mathematical situations and structures using symbols
3. Analyse mathematical situations using algebraic thinking
4. Use models to represent, analyse, and interpret quantitative relationships
5. Understand, interpret, and use change in a variety of situations

Geometry

1. Use knowledge of characteristics and properties of shapes and solids to express mathematical ideas, develop arguments and form and analyse spatial and geometric relationships
2. Specify locations using a system of coordinates
3. Make inferences about spatial relationships using coordinate geometry
4. Analyse mathematical situations using symmetry and transformations of space
5. Use mental visualization to solve problems
6. Use spatial reasoning to analyse mathematical situations and solve problems
7. Use geometrical modelling to solve problems

Measurement

1. Understand measurable attributes of objects
2. Understand units and systems of units
3. Understand and apply appropriate processes and techniques of measurement
4. Use appropriate tools and formulae to determine measurements

Data Analysis and Probability

1. Formulate, frame, and test hypotheses with data
2. Organize and reorganize data to make inferences and draw conclusions
3. Collect appropriate data for specific purposes
4. Select and use appropriate statistical techniques to analyse data
5. Use data to make predictions, to distinguish fact from opinion and to show relationships
6. Understand risk and chance

Processes/Problem - Solving

1. Build new knowledge through problem solving
2. Solve a variety of problems in a number of contexts
3. Apply and adapt a variety of appropriate strategies with the flexibility to solve problems
4. Understand risk and chance

Reasoning and Proof

1. Recognize and understand reasoning and proof as critical in communicating mathematical ideas
2. Make and investigate mathematical conjectures
3. Develop and evaluate mathematical arguments and proofs
4. Use different types of reasoning
5. Prove by different methods

Communication

1. Explore, reflect on, and express their mathematical thinking
2. Present their mathematical thinking coherently and clearly to a variety of audiences
3. Analyse and evaluate mathematical thinking of others
4. Use precise mathematical language to express ideas

Connections

1. Recognize and use connections among mathematical ideas
2. Recognize and use mathematics in a variety of contexts

Representation

1. Use representations to express mathematical ideas
2. Solve problems effectively using mathematical representations
3. Model and interpret physical, social, and mathematical phenomenon using representations

Articulation of the National Curriculum Outcomes with NCTM Outcomes

Essential Learning Outcomes for each module are also aligned with some of the NCTM outcomes. However, these outcomes are aligned with the vision and mission of the national agenda.

Abbreviations used for ELO's

Essential Learning Outcome	Designated Symbol
Aesthetic Expression	AE
Communication	C
Citizenship	CIT
Personal Development	PD
Problem Solving	PS
Technological Competence	TC

MATHEMATICS

MODULE 1



EXPLORING WITH NUMBERS

MODULE 1: EXPLORING WITH NUMBERS

This module includes:

Rationale	2–M1–1
Aim	2–M1–1
Goals	2–M1–1

Unit 1 Understanding numbers

General Outcomes	2–M1–2
Specific Outcomes by Levels	2–M1–2
- Core	2–M1–2
- Extensions	2–M1–3
Content	2–M1–4
Instructional Strategies/Methods	2–M1–4
Suggestions for Assessment	2–M1–5
Resources	2–M1–5
Suggested Teaching and Learning Activities	2–M1–6

Unit 2 Numbers in 2-dimensions

General Outcomes	2–M1–7
Specific Outcomes by Levels	2–M1–7
- Core	2–M1–7
- Extensions	2–M1–8
Content	2–M1–8
Instructional Strategies/Methods	2–M1–9
Suggestions for Assessment	2–M1–9
Resources	2–M1–9
Suggested Teaching and Learning Activities	2–M1–10

Unit 3 Working with vectors

	2–M1–12
General Outcomes	2–M1–12
Specific Outcomes by Levels	2–M1–12
- Core	2–M1–12
- Extensions	2–M1–13
Content	2–M1–13
Instructional Strategies/Methods	2–M1–14
Suggestions for Assessment	2–M1–14
Resources	2–M1–14
Suggested Teaching and Learning Activities	2–M1–15
Sample Lesson	2–M1–16
Standards	2–M1–21

EXPLORING WITH NUMBERS

Rationale

Mathematics is an essential ingredient in understanding and mastering the real world, the economy, technology and various other aspects of the educated world. Number pervades all aspects of Mathematics. It is critical that all students develop a command for the use of numbers. ‘Exploring with Numbers’ offers students the opportunity to understand numbers, number systems, number relationships and ways of representing numbers both as scalar quantities and vector quantities. It enables students to organize and manipulate data using matrices. In addition, students are provided with the opportunity to develop algebraic thinking skills, logical-mathematical thinking skills and problem- solving skills, through the use of numbers.

Aim

This module is designed to develop aspects of problem - solving, communication, personal development and technological competence, through the use of content in Number Theory including Matrices and Vectors and Algebra. It is presented in three (3), Units, each of which will focus on different uses and representations of number in a variety of contexts.

Goals

This module supports the attainment of outcomes in which students will:

1. enhance logical and strategic thinking using numbers and number relationship (C, PS);
2. develop creativity, enhance critical thinking, and the spirit of enquiry using number relationships, number patterns, symbols and representations (PS, C, CIT);
3. develop an appreciation for the widespread application of numbers in different applied fields such as Engineering, Science, Business, consumerism and in everyday life (PS, PD);
4. appreciate the clarity and preciseness of communicating ideas through number and symbols (C, AE, PD);
5. appreciate the use of number in different cultures and societies (PD, CIT).

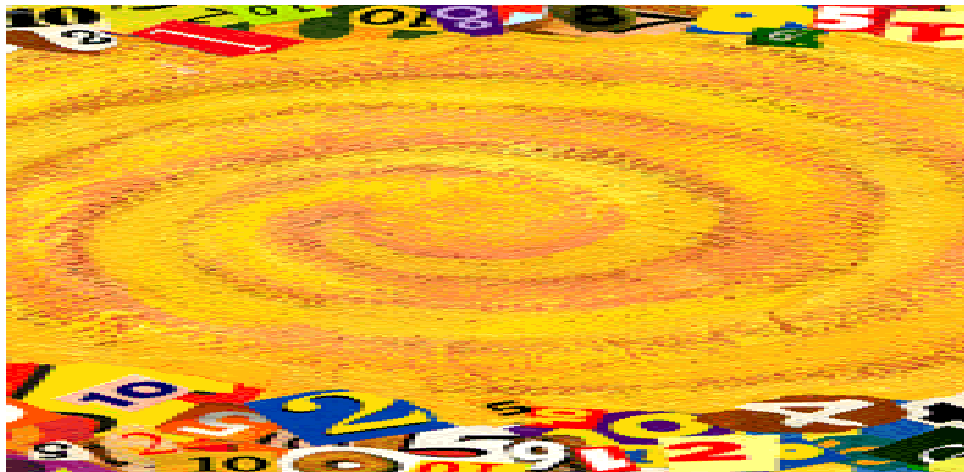
This module comprises three units:

Unit 1 – Understanding numbers

Unit 2 – Numbers in 2-Dimensions

Unit 3 – Working with vectors

UNIT 1



UNIT 1 – UNDERSTANDING NUMBERS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate an understanding of the concept of number and explore the properties, representation and interrelationships among Number Systems (PS, PD, C);
2. use number concepts to model and solve real-world problems (PS, PD, TC, C);
3. demonstrate deep understanding and appreciation of large and small numbers and their representations. e.g. scientific notation and exponential form (PS, PD, C, TC);
4. develop fluency and accuracy in performing operations with real numbers of any size, with and without the use of calculators (PS, PD, TC);
5. demonstrate algebraic thinking skills by using numbers, number patterns, representations and mental models to describe and solve problems (C, PS).

SPECIFIC OUTCOMES:

CORE:

Students are expected to:

1. compare and contrast the properties of number sets including the rational, irrational and real number sets e.g. inclusion property $N \subset W \subset Z \subset Q \subset R$;
2. use Venn diagrams to solve real world problems involving up to three sets;

3. perform basic operations involving numerals in base 2 (including the strategy of changing bases);
4. perform computational tasks using number properties and the laws for number operations (binary and mixed);
5. represent and perform computational tasks on numbers in any form, with and without the use of calculators;
6. use numeric and symbolic representations to explain patterns and make inferences;
7. express relations between odd and even numbers using simple statements such as:
The square of an even number is even,
1 added to the square of an even number is odd,
 $x^2 + 1$ is odd if x is even,
if $m * n$ means m^n then $m * n$ is even if m is even, and so on;
8. solve simple problems involving everyday business transactions and consumerism. (including the computation of ratios, rates, percentages, sales, purchase tax, invoices, shopping bills, utility bills, exchange rates and taxes, simple and compound interests, investments, hire purchase, mortgages and appreciation/depreciation.);
9. make estimates and check computations to determine and confirm the appropriateness of the results of operations performed on numbers of any size.

EXTENSIONS:

Students are expected to:

10. *perform basic operations on numbers of any size;*
11. *model and solve problems on natural growth and decay;*
12. *estimate errors in measurements and determine levels of accuracy in measures and calculations;*
13. *perform basic operations involving numerals in any base, with or without changing the base;*
14. *use numbers expressed in exponential form to represent the end points and the width of intervals;*
15. *write simple proofs based on number properties and laws; Use convincing arguments to using statements in symbolic form*
(e.g. prove that $x^2 < x$, $\forall x : -1 < x < 1$;
16. *use symbolic form to make simple statements and draw basic conclusions about number properties and conjectures;*

17. *model simple real life phenomena involving rate of increase or decrease using exponential graphs;*
18. *sketch a family of curves to represent functions of the type $y = a^x$, where $a \in \mathbb{Z}$, and show how the curves relate to each another.*
-

Content

- ❖ Number concepts and properties of numbers
 - Factors, Multiples, LCM, HCF
 - Laws of indices
 - Squares and cubic numbers; odd and even numbers etc.
 - Additive and multiplicative identities and inverse
 - Commutative, Distributive, Associative Laws
 - Concept of closure
 - Properties of zero
- ❖ Real number system – Natural numbers, Whole Numbers, Integers, Rational Numbers
- ❖ Irrational Numbers
- ❖ Inclusion relations $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$
- ❖ Number Bases
- ❖ Order of operations – mixed operations, binary operations
- ❖ Sequences
- ❖ Metric conversions – use of metric conversion scales, 12-hour and 24-hour clock
- ❖ Significant figures
- ❖ Scientific notation/Standard form
- ❖ Exponential form including writing intervals using exponential form
- ❖ Consumer Arithmetic

Instructional Strategies/Methods

- Activate prior knowledge
- Presentations
- Research, investigations and discussions
- Worksheets

- Web based activities
- Explorations/Investigations
- Teacher/student demonstrations
- Use of workstations
- Games

Suggestions for Assessment

- ✓ Pencil and paper tests
- ✓ Quizzes and games –Competition
- ✓ Teacher observation
- ✓ Peer observation
- ✓ Student Project/Group Project
- ✓ Worksheets
- ✓ Simulation
- ✓ Portfolios
- ✓ Performance tasks
- ✓ Presentations – individual, group
- ✓ Student Self-Assessment

Resources

- Computers
- Internet
- Calculators
- White board
- Manipulatives
- Games
- Flash cards
- Play money

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

1. Investigating Numbers

Students will:

1. use calculators or computers to generate whole numbers or number patterns using additions of 2 or 5 or other quantities;
2. explore and present ways to count or derive the number of numbers within two limits.

2. Exploring the commutative, distributive and associative laws

Students will:

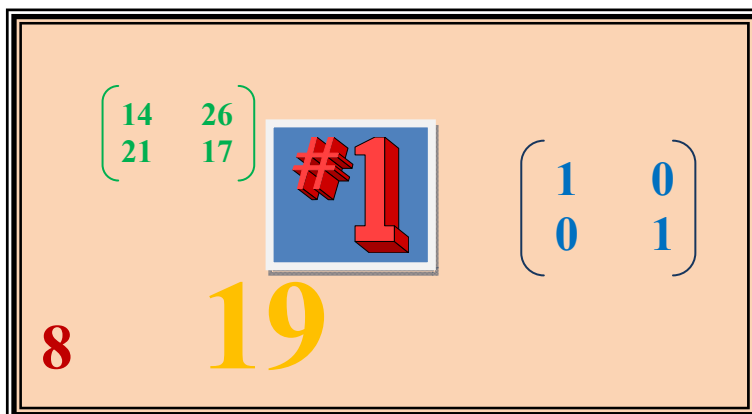
1. use calculators to perform computations such as:
 $5 + 2$, $2 + 5$, $5 \div 2$, $2 \div 5$, $5 + (2 + 1)$, $5 \times (2 + 1)$, and so on changing operations between numbers, noting when the exact results are obtained;
2. explain patterns and replace numbers by letters to come up with generalizations for discussion. (Students should be encouraged to investigate whether certain laws hold for all types of numbers and explain the situations in which the laws do not hold.).

3. Experimenting with numbers

Students will:

1. work with manipulatives at workstations building squares and different sets of numbers including odd, even, square and composite numbers. They will present their findings for discussion;
 2. investigate number properties and relations using a straight object to represent the number line. (Markers or pins could highlight different points on the line. Students could use calculators to perform different computations in their investigations and position other pins or markers based on their results.).
-

UNIT 2



UNIT 2 –NUMBERS IN 2-DIMENSIONS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding that a matrix is a number system that has some properties of the real number system (AE, PD);
 2. use matrices to represent and manipulate data (AE, PS);
 3. model and solve systems of linear equations using matrices (PS);
 4. appreciate the practical use of matrices to model real-world situations (AE, PD);
 5. use matrices to solve real-world problems (PS, CIT).
-

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. arrange numbers and objects in rows and columns based on given criterion;
2. organize and reorganize numerical information using matrices;
3. use matrix terminology (such as order, inverse, transpose, identity, null matrix, identity matrix, adjoint, elements, rows and columns) to explain and solve problems;
4. perform addition, subtraction, and multiplication of matrices (including both scalar and matrix multiplication);
5. determine whether a matrix format is appropriate for presenting or processing information in any given situation;

6. make inferences from data presented in matrix form;
 7. determine whether given sets of matrices are comparable for performing particular operations;
 8. calculate the determinant of a 2×2 matrix;
 9. use algebraic thinking to solve problems involving equal, equivalent and singular matrices;
 10. compute inverses of 2×2 non-singular matrices;
 11. use matrices to solve systems of 2 linear equations;
 12. transpose a matrix;
 13. solve simple real-life problems using matrices up to order 2×2 ;
 14. simulate matrices to present and analyze data extracted or collected from familiar sources encountered in everyday interactions. (e.g. matrices up to order 3×3 in utility bills, advertisements, shopping bills and invoices);
-

EXTENSIONS

Students are expected to:

15. *solve simple matrix equations of the form $A + BX = C$ where A and B are 2×2 and X and C are 2×1 matrices;*
 16. *add, subtract and multiply matrices with more than 2 rows and or two columns;*
 17. *use electronic technology to perform operations on matrices;*
 18. *solve problems in which the matrix equation $A + BX = C$ could be used as a model.*
-

Content

- ❖ Concept and properties of a matrix
 - row, column, types of matrices, order, unit and zero matrices, determinant of a matrix, singular, non-singular, transpose, equal etc.;
- ❖ Matrix operations
 - compatibility for multiplication, order of matrices not exceeding 3×3
- ❖ Inverse of 2×2 matrices – matrix method;
- ❖ Modelling simple real life situations using matrices up to and including 3×3 matrices.

Instructional Strategies/Methods

- Student led activities
- Teacher guided activities
- Investigations
- Discussion
- Worksheets
- Web-based activities
- Project-based activities
- Demonstrations
- Student research
- Data collection activities
- Computer-aided activities

Suggestions for Assessment

- ✓ Peer evaluation
- ✓ Worksheets
- ✓ Reports
- ✓ Pencil and paper tests
- ✓ Projects.
- ✓ Performance tasks.
- ✓ Presentations.

Resources

- Whiteboard
- Presentation hardware
- Internet
- Charts
- Worksheets
- Computers

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

1. Students' spending

Students will:

1. use their previous knowledge and experience with numbers to classify data in rows and columns based on some criterion.

For example, number of dollars spent on a daily basis in one week during the morning and afternoon sessions while at school could be presented as:

	Mon	Tue	Wed	Thu	Fri
a.m.	2	5	6	1	8
p.m.	3	10	18	1	20

2. present their data and share their experiences through guided class discussions.

N.B.: The teacher will guide the discussions appropriately so as to address the sensitivities of students' socio-economic status and the financial challenges that they may be facing. Examples to be used in this exercise must be carefully chosen.

2. Newspaper readership

Students will:

1. examine newspaper readership among men, women and children.

Data is collected on the number of different people including men, women and children who read the daily newspapers. Analysis of the data may involve, among other things, computation of total readership or computation of products as in revenue from sales. The use of matrices to display the data is recommended.

3. Investigating operations with matrices

Students will:

1. discover that only matrices of the same order could be added or subtracted.

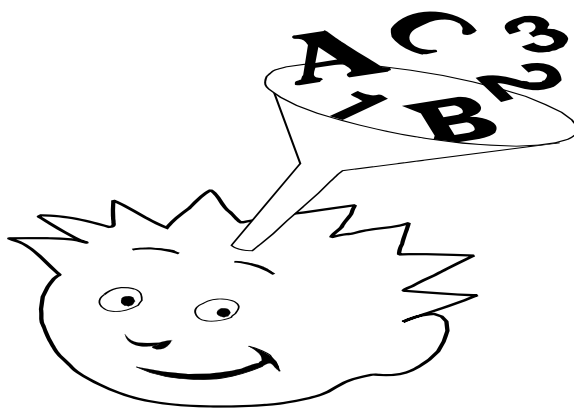
Students could be presented, for example, with situations leading to two matrices such as;

$$(5 \ 6 \ 8) \text{ and } \begin{pmatrix} 4 & 2 \\ 1 & 6 \end{pmatrix}$$

to find the sum. They should then see the problem involved;

2. make comparisons between operations on systems of numbers and on systems of matrices to investigate similarities and differences.
For example students may investigate whether the commutative law holds for multiplication of matrices.
-

UNIT 3



UNIT 3 – WORKING WITH VECTORS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding that vectors belong to number systems that share some properties of the real number system (AE, PD, PS);
 2. demonstrate understanding of vectors, their properties and applications (PD, PS);
 3. demonstrate enhanced geometric thinking, analytical and technological skills (PS, TC);
 4. show an appreciation of the use of vectors in analyzing, interpreting and communicating in spatial context (AE, C, PD, PS);
 6. show increased ability to apply new technologies to deepen understanding of space (PD, TC);
 7. show appreciation of the preciseness of vectors as an analytical tool for modelling and solving real-world problems (PD, PS).
-

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. use the concepts associated with vectors appropriately e.g. scalar (magnitude) and direction;

2. use knowledge of lengths and directions to describe location of places, points or positions in two-dimensional space using a point and axes of reference (e.g. describe the location of one's house relative to school as x km towards the north and y km towards the east where x and y are numbers);
 3. represent a vector as a column matrix;
 4. perform basic operations involving vectors using vector algebra (including addition, subtraction, multiplication by a scalar - integers and rational numbers);
 6. demonstrate understanding of equal vectors, parallel vectors, displacement vectors, and the inverse of a vector;
 7. represent vectors, parallel vectors and displacement vectors on the Cartesian plane (coordinate system in two dimensions only);
 8. determine the magnitude (modulus) of a vector;
 9. obtain the resultant of two vectors using vector algebra or geometrical drawing – Triangle and Parallelogram laws;
 10. use a vector approach to solve problems involving vectors;
 11. describe displacements, velocities, and forces using correct vector terminology;
 12. model displacements, velocities and forces as vectors using spatial representations and simulations;
 13. stimulate interest in exploring simple vector applications by modelling and solving real-life problems.
-

EXTENSIONS

Students are expected to:

14. *use position vectors to locate and describe points in two dimensions;*
 15. *determine the unit vector in a given direction;*
 16. *use vectors to model practical situations;*
 17. *apply vector methods to solve problems in the real - world;*
 18. *make informed statements on the use of vectors in analyzing real - life problems as opposed to use of number only.*
-

Content

- ❖ Concept of a vector – magnitude (scalar), direction;
- ❖ Properties of vectors (modulus, equality, equivalence, inverse of a vector);
- ❖ Vector representations (geometric, algebraic);

- ❖ Vector operations (addition, subtraction, multiplication by a scalar).
- ❖ Position vectors
- ❖ Unit vectors
- ❖ Vector Methods

Instructional Strategies/Methods

- Class discussion
- Socratic questioning
- Practical explorations/investigations
- Research, investigations and discussions
- Teacher/student demonstrations
- Use of workstations
- Worksheets
- Web based activities

Suggestions for Assessment

- ✓ Paper and pencil tests
- ✓ Problem sheets
- ✓ Journals
- ✓ Performance tasks
- ✓ Project work

Resources

- Computer lab
- Internet
- Manipulatives
- Multimedia projector
- Navigational software

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

Working with vectors

Teachers will:

1. introduce this unit in a very practical, activity-oriented way. Boys, in particular, have fun experiences in the use of vector related in sometimes-daily activity in computer games. All lessons could be centred on such an activity;
2. lead students to make presentations in which properties and operations of vectors are simulated and understood without using the technical terms too early;
3. use web-based resources or navigational software to facilitate student learning;
4. introduce more complex terminology as students gain confidence and develop skills in working with vectors.

Students will:

1. engage in everyday activity in which they are involved on a daily basis;
 2. use their knowledge and understanding of number to explore systems of vectors;
 3. appreciate that vectors is a means of working simultaneously with magnitude and direction to analyze systems;
 4. engage in practical work on velocities, displacements and forces to gain a deeper understanding of vectors. If available, computer graphics could be used;
 5. model movements of particles using simulations.
-

SAMPLE LESSON #1

Module 1: Unit 1

Number Operation

General Topic: Number Operation

Learning Objectives: Students will be able to:

- make estimates about real-life data
- collect and analyze data
- represent data through physical and graphical means
- develop number sense

Materials: Stopwatches or clocks with second hand

- Adding machine tape
- Scissors
- Calculators
- **Every Breath You Take Activity Sheet**

Introduction

Begin the investigation by encouraging students to give an example of something that they do once a day such as eat breakfast, listen to school announcements or play with a friend. Ask them to name something they do about ten times each day, which might include saying hello in the hall, changing the television channel, or writing their names on papers. Finally ask students to name something that they do at least one hundred times a day. A narrow range of responses often includes blinking and breathing. If not, take a deep breath and ask, "**What about breathing?**"

Ask students, "Do you take more than 100 breaths in a day? More than 100 or 1000 breaths in an hour?" After a brief discussion, distribute the Every Breath You Take Activity Sheet and ask each student to estimate the number of breaths a person takes each hour. Have each student write down an estimate.

TEACHER ACTIVITY	STUDENT ACTIVITY
<ul style="list-style-type: none"> • Ask the students to state their estimates one by one so that the class can record them on the board. • Teacher groups students in pairs to answer questions 3 and 4. • Teacher asks pupils to arrange their answers in ascending order and asks pupils to comment on the results. • Encourage students to explore how they might use their estimates of the number of breaths taken in one hour to estimate the number taken in a day. • Teacher leads off discussion as to why that estimate may be different from the actual number of breaths taken in a day. • To judge the reasonableness of student's estimates, ask, "How could you find out how many breaths a person actually takes in a day?" • Have students implement their strategies and compare their answer with one another. 	<ul style="list-style-type: none"> • Students state their estimates. • Students work in pairs and after recording their findings they comment on their results. • Students make suggestions and comment on suggestions made by their peers. • Students participate in the discussion. • Allow students to brainstorm solving the problem. They might work with a partner and count the number of breaths in a given period of time—for instance, one minute—or determine the time it takes to breathe a given number of times, say, one hundred. • Students compare their answers.

Conclusion

Conclude the investigation by encouraging students to look for other things that are done many times every day in their world. By counting the number of times that we do these things, a sense of large numbers and a familiarity with them may be established.

Extension

From the information identified above, determine a class average of the number of breaths taken per hour.

SAMPLE LESSON #2 - PERFORMANCE TASK

Module 1 - Unit 3

Vectors: Triangle Law of Vectors

Part 1:

Explain how the hovercraft works.

Part 2:

A hovercraft starts at $(0, 0)$ and makes three parts of a flight represented by the following vectors in this order: $(7, 4)$, $(3, 2)$ and $(-1, 6)$.

If an airplane starts at $(0, 0)$ and flies the same three parts in a different order, will it end up at the same ending point?

Overview

The task is a demonstration of Multiple Intelligences in the classroom and the development of Media Literacy skills in students. The teacher's role in this activity is to facilitate varied modes of instructions, a variety of resources, media and other support needed by students, a sample of students' work expected by the teacher with accompanying Assessment Scoring Rubric for assessing students' work. Students will select the mode of Intelligence they wish to adopt for the task and the resources required for accomplishing the task. The teacher will provide media support required for the Internet research and also, at the end of the lesson, to facilitate the creation of a poster or a Power Point presentation of what was learnt.

Learning Outcomes

Students will be able to:

1. research and explain how the hovercraft works;
2. represent vectors in two-dimensions;
3. apply the Triangle Law of Vectors to determine the result of two or more vectors inclined at an angle to each other;
4. apply new technologies to simulate, model and solve simple real life problems involving displacements;
5. appreciate the usefulness of new technologies to deepen spatial awareness.

Prior Knowledge for students:

1. Students should be able to use a web browser to access Internet sites.
2. Students should have some prior experience with Navigational Software.
3. Students should know how to graph points in the coordinate plane.

Prerequisite Knowledge for teacher:

1. Teacher should be able to use a web browser to access Internet sites.
2. Teacher should have some prior experience with Power Point and Navigational Software.

Time Frame: Two 40-minute periods

Materials: Student

- Computers with web browser and Navigational Software installed
- Grid Paper
- Plane Paper
- Pencil
- Ruler
- Manipulatives
- Pair of Scissors
- Glue
- Pair of compasses.
- Coloured Pencils/Markers

Materials: Teacher

- Computer with web browser and Navigational Software installed
- Assessment Scoring Rubric
- Multimedia Projector

Teacher Instructions

- Whole group – Brainstorming ideas – strategy for adding two or more vectors inclined at an angle to each other leading up to head-to- tail alignment. Teacher establishes **Triangle Law of Vectors**.
- Whole Group: Demonstration of Triangle Law of Vectors using diagram and multi-media.

Student Activities

- Small group - Exploration of Resultant Vector using notes copied from teacher's demonstration and resources of choice
- Individual - direct observation by teacher
- Small group - Internet research on hovercraft and how it works
- Small Group - Multiple Intelligences approaches to simulation/creation of solution to Performance Task such as:
 - paper cut outs of vectors using plane paper;
 - outdoor games using wooden stakes to represent given co-ordinates and pacing; marching, dancing, walking to obtain equivalent path of resultant vector;
 - locating given co-ordinates on Cartesian Plane drawn on grid paper;
 - analytical method of scale drawing using pair of compasses and rulers
 - computer simulation of vectors
- Small group - Oral presentation comprising a poster, a Power Point presentation or presentations modelled after Multiple Intelligences formats.

Extension

- Have students extend the task to the addition of three or more vectors using quadrilaterals or other polygons to obtain resultant vector.

Assessment

- Formative Teacher Observation of groups at work
 - Oral presentation of Multiple Intelligences presentations and extension work where applicable
-

STANDARDS

MODULE 1

Mathematics Standards

MODULE 1: EXPLORING WITH NUMBERS

At the end of Form 5, students will:

1. Understand Numbers, ways of representing numbers, relationships among numbers and Number Systems

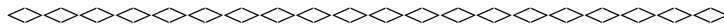
- 1.1 Model and represent exponential change graphically including the use of computer software;
- 1.2 Represent real numbers, including large and small numbers, in exponential form and standard form (Scientific notation);
- 1.3 Formulate simple worded statements about relationships among number systems and present simple logical mathematical statements to prove or refute them, and convert numeral expressed in one base to another base up to and including base ten (10);
- 1.4 Use algebraic and geometric reasoning to represent and recognize relationships in number patterns;
- 1.5 Model and represent real world problems using real number sets up to and including three (3) intersecting sets;
- 1.6 Represent numbers in equivalent forms (integers, rational numbers, irrational numbers and percent).

2. Understand meanings of operations, how they relate to one another and perform operations on numbers, matrices and vectors

- 2.1 Determine the resultant of two vectors using an algebraic or vector method;
- 2.2 Determine the modulus (length) of a vector;
- 2.3 Explain the effect of multiplying a vector by a scalar;
- 2.4 Perform with fluency the operations of addition, subtraction, multiplication on matrices including multiplying matrices by a scalar and determine the determinant, adjoint, inverses and transposition of vectors;
- 2.5 Perform basic operations $+$, $-$, \times , \div on numbers expressed in exponential form, including situations where the exponential form is not changed;
- 2.6 Perform basic operations $+$, $-$, \times , \div on real numbers including, large and small numbers, using mental computation, paper-and-pencil calculations for simple cases and technological tools for more complicated cases.

3. Compute fluently and make reasonable estimates.

- 3.1 Compute fluently performance tasks involving binary operations;
- 3.2 Compute fluently percentages / percentage increase/percentage decrease of quantities e.g. profit, discount, value added tax, invoices, loans and investments, Hire Purchase, Mortgages, Income tax;
- 3.3 Compute fluently performance tasks involving ratios of quantities and the unitary method e.g. currency conversions;
- 3.4 Compute fluently performance tasks involving rates and taxes e.g., utility bills, wage;
- 3.5 Make quick rough estimates of quantities in everyday life e.g., budgets, shopping bills, returns on investments, value added tax etc.



MATHEMATICS

MODULE 2



USING THE EVIDENCE

MODULE 2: USING THE EVIDENCE

This module includes:

Rationale	2–M2–1
Aim	2–M2–1
Goals	2–M2–1

Unit 1 From data to information

General Outcomes	2–M2–3
Specific Outcomes by Levels	2–M2–4
- Core	2–M2–4
- Extensions	2–M2–4
Content	2–M2–5
Instructional Strategies/Methods	2–M2–5
Suggestions for Assessment	2–M2–5
Resources	2–M2–6
Suggested Teaching and Learning Activities	2–M2–6

Unit 2 Making Informed decisions

General Outcomes	2–M2–7
Specific Outcomes by Levels	2–M2–8
- Core	2–M2–8
- Extensions	2–M2–8
Content	2–M2–9
Instructional Strategies/Methods	2–M2–9
Suggestions for Assessment	2–M2–10
Resources	2–M2–10
Suggested Teaching and Learning Activities	2–M2–11

Sample Lesson	2–M2–12
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Standards	2–M2–15
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USING THE EVIDENCE

Rationale

The new millennium is characterized by new and innovative ways of dealing with data and managing information. Technological advances have supported this trend as it is now possible to draw on newer and more efficient tools and techniques to collect and manipulate data in facilitating informed decision-making. In this regard, it is therefore necessary to develop in students the requisite knowledge and skills to examine and use data for a variety of reasons and not just for ethical considerations.

‘Using the Evidence’ helps to develop the skill sets and conceptual knowledge that will lay the foundation for a more comprehensive understanding of data analysis and how to use data as evidence to support statements, validate proofs and seek to expose basic truths.

Aim

This Curriculum integrates appropriate technologies into the learning and practice of Mathematics, while recognizing the continuing importance of students’ mastery of essential numerical and algebraic skills. This module aims to facilitate aspects of Personal development, Problem-solving, Communication and Technological Competence, as well as Aesthetic Expression and Citizenship, through the use of content in Statistics. Through this module, students will be guided and motivated to develop statistical awareness and become self-directed learners.

Goals

This module will support the attainment of outcomes in which students:

1. develop dispositions to seek clear and unambiguous interpretations of the real world through the use of relevant data and appropriate analytical tools and techniques. (TC,C, PS);
2. develop appreciation for the random nature of many situations encountered in daily life and the need to make adjustments to cater for this. (PD);
3. are stimulated to research and acquire large amounts of data to test hypotheses instead of limiting themselves to a few sources only and very few data points. (PS, PD);
4. acquire enhanced problem-solving skills through development of data management skills. (PS, PD,C, AE);
5. are expected to become familiar with popular software used in Data Processing, for example, Microsoft Word, Microsoft EXCEL, Microsoft Access, Power Point and other office products. (AE, PD, TC, PS);

6. acquire the ability to make more informed decisions in matters of finance, job selections and university selections among many other areas where decisions are made based on the collection of large amounts of data. (AE, CIT);
7. develop an appreciation of the use of data in interpreting and understanding systems. (PS);
8. develop skills of accessing real life data and the ability to verify authenticity and reliability. (C, CIT, PD, TC).

This module comprises two units:

Unit 1 – From data to information

Unit 2 – Making informed decisions

UNIT: 1



UNIT 1 – FROM DATA TO INFORMATION

GENERAL OUTCOMES

Students are expected to:

1. demonstrate a basic understanding of concepts of data collection, presentation and interpretation (AE, C, PD,);
2. demonstrate an appreciation of ethical issues as they relate to the use of data (CIT, PD);
3. demonstrate enhanced knowledge and skills in the management and interpretation of data (C, PD, PS);
4. demonstrate improved skills in the use of modern technology in analyzing and presenting data (C, PD, PS);
5. demonstrate confidence in communicating the results of data gathering and processing (C, PS);
6. appreciate the weaknesses in representing data with the different forms of representations possible (C, PD);
7. appreciate the difference between primary and secondary sources of information (PD).

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. use appropriate methods to collect data;
2. apply skills of tallying, tabulating and graphing to present data;
3. identify basic ethical violations in collecting and using data from existing sources;
4. use Bar Charts, Histograms, Frequency Polygons, cumulative frequency curves appropriately to present data;
5. use a basic software package to sort, summarize, and present data;
6. extract and analyse data from everyday sources such as newspapers, to use as evidence to support simple claims;
7. distinguish between primary and secondary sources of data;
8. distinguish between quantitative and qualitative data;
9. write simple statements to explain the terms *sample* and *population*;
10. identify and explain bias in data collected for a particular purpose (For example bias in data from a manufacturing company about the benefits of its product);
11. distinguish between reputable and non-reputable sources of information on the Internet and World Wide Web.

EXTENSIONS

Students are expected to:

12. *make inferences based on statistical data collected;*
 13. *use suitable evidence to evaluate claims and conclusions made in reports, articles, news items, studies and statements made by individuals for validity and bias;*
 14. *identify and explain possible reasons for deliberate misrepresentation of data in given situations;*
 15. *describe ways of drawing random samples.*
-

Content

- ❖ Data and information
- ❖ Collecting data
 - Data collection instruments
- ❖ Grouped and ungrouped data
- ❖ Types of data:
 - Continuous and discrete
 - Qualitative and quantitative
- ❖ Bias in data
- ❖ Data sources
- ❖ Representing data e.g. Tally charts, Frequency tables
- ❖ Graphing data : Pie chart, Bar charts, Line graphs, Histograms
- ❖ Using data, tables and diagrams

Instructional Strategies/Methods

- Activate prior knowledge
- Presentations
- Research, investigations and discussions
- Worksheets
- Web based activities
- Explorations/Investigations
- Teacher/student demonstrations
- Use of workstations
- Games

Suggestions for Assessment

- ✓ Debates
- ✓ Journals
- ✓ Paper and pencil tests
- ✓ Performance tasks
- ✓ Portfolios
- ✓ Problem sheets
- ✓ Project work

Resources

- Internet
- Bulletin, books, brochures on ethics and research
- Resource Person/Expert
- Manipulatives
- Multimedia projector
- Statistical soft ware

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

Data -driven understanding versus Hearsay

Teacher will:

1. encourage students through group discussions to examine the value of information that may be acquired indirectly i.e., secondhand information, hearsay, rumours;
2. provide inputs and cases for considerations. Special cases of reputations of individuals being targeted by individuals with nothing but hearsay evidence and rumours to base their facts on;

The teacher may give inputs via facts of such cases which might be drawn from the areas of politics, business, local village, etc;

3. demonstrate independence of the result of any game of chance, (example the toss of a fair coin or drawing via replacement of a tennis ball in games of chance machines) from the result of any previous toss or draw.

Students will:

1. participate in group discussions to discuss the likelihood of predicting the results of games of chance based on dreams, signs and such like;
2. make entries in their journals as to their ideas and points of interest;
3. suggest other situations where data is used to guide decision-making.

N.B.: This activity may be repeated using different examples and contexts to facilitate informed decision-making as presented in Unit 2 of this module.

UNIT: 2



UNIT 2 – MAKING INFORMED DECISIONS

GENERAL OUTCOMES

Students are expected to:

1. develop analytical skills in the use of empirical data to make inferences and draw conclusions (PD, PS);
2. develop an appreciation of the role of assumptions in modelling (PD);
3. develop a basic understanding of the use of models to make predictions and the limitations to which they may be subjected (PD, PS);
4. demonstrate understanding of the difference between theoretical and experimental probability (C, PD);
5. develop appropriate use of measures of central tendency in social and economic contexts (PD, PS);
6. use statistical data to assist in making informed decisions (C, PD, PS).

SPECIFIC OUTCOMES

CORE

Students are expected to:

1. determine the frequency and relative frequency from data sets in organized or in raw form;
 2. distinguish between mean, mode and median;
 3. determine the mean, mode and median for any data set including data in grouped frequency tables and stem and leaf plots by observation or calculation;
 4. determine the upper quartile and lower quartile in a discrete data set;
 5. make and evaluate inferences about one or more data sets based on the values of mean, mode, median, range, deviations and quartiles;
 6. construct frequency polygons, histograms and cumulative frequency polygons and curves and use them to accurately present ideas and information, taking their limitations into consideration;
 7. use theoretical probability to explain and evaluate the risk involved in making simple decisions in real - life situations;
 8. correctly use frequency polygons and histograms to present information;
 9. use statistical features on an electronic calculator to do basic computation of mean and standard deviation;
 10. compute theoretical probability in simple statistical experiments with two (2) dice and two (2) or three (3) coins;
 11. relate the concepts of outliers or extreme values to the mean and data values in the set;
 12. compute and explain range and deviations for discrete data.
-

EXTENSIONS

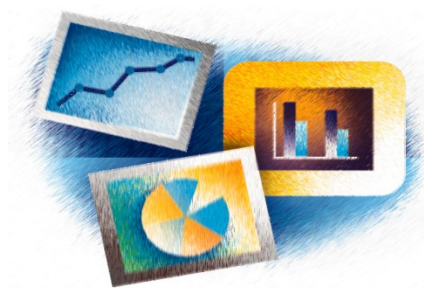
Students are expected to:

13. *explain limitations in the use of polygons and histograms to represent data;*
14. *use electronic instruments to input, display and do simple computations on small discrete data sets;*
15. *identify basic ethical violations in data use, data collection and data presentation in simple everyday situation;*
16. *distinguish between mutually exclusive and independent events and use the properties to calculate probability in simple statistical experiments;*

17. *compute conditional probability in simple two (2) dice experiments;*
18. *make connections with statistical information and real-world contexts to facilitate informed decision-making.*
-

Content

- ❖ Frequency polygons
- ❖ Cumulative frequency polygons
- ❖ Sampling
- ❖ Computation of arithmetic mean and standard deviation using programmed features of calculators
- ❖ Mode and Median
- ❖ Mode and modal class
- ❖ Range
- ❖ Quartiles
- ❖ Range and Deviations
- ❖ Hypothesis formulation
- ❖ Procedures of a statistical investigation
- ❖ Probability Tree diagrams
- ❖ Codes, Bar Codes, Braille, Postcode, telephone numbers



Instructional Strategies/Methods

- Cooperative learning groups
- Role play/ simulation
- Project-based activities
- Research
- Presentations
- Demonstration
- Simulations
- Blogs
- Explorations
- Spreadsheets in computer laboratory
- Socratic questioning
- Video clips

- Clips from Internet, magazines, existing studies
- Guided discussion
- Cooperative groups
- Computer- based instruction
- Use of video clips
- Field trip.
- Role play/classroom theatre
- Internet facilities
- Use of board games

Suggestions for Assessment

- ✓ Portfolios showing checklists for interviews, sample questionnaires, pictures and notes
- ✓ Journals
- ✓ Assessment rubrics
- ✓ Pen and paper test
- ✓ Presentations (oral and written)
- ✓ Reports

Resources

- Internet
- Camcorders
- Tape recorders
- Digital cameras
- Video/ DVD
- Pictures
- Multimedia projector
- Worksheets
- Multimedia including: document projectors, screen, various media players for audio visual effects
- Calculators
- Graphing software
- Portfolio showing various distributions of data collected and organized by the student

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

This unit could be based around a project in which students learn to collect, analyze and present data to solve a problem, do an investigation or study or use statistics in some other real - life or simulated situation. Activities could involve the use of workstations at which students acquire real life experiences.

1. Mock Elections

As an example, the teacher could hold a mock election with all the features of a real election, including taking polls, making predictions and using data in all the ways in which it is used in a real election. The entire classroom could be converted to a centre with workstations at which students acquire knowledge, skills, and understanding in this unit.

2. Generating and Using Data

Ask students to do research, gather or simulate data in one or more projects. For instance, bias, sample, population, sample mean and population mean could become real if students use a one-year group, say, as a population. The age of all students could be obtained from school records and a mean found. This mean could be compared with the mean of samples taken from the population in different ways such as through asking for volunteers to give their age. Discussions could be held to explain problems with the process.

The teacher can use a calculator to generate random numbers and code each student so that a random sample can be taken from the class. The students are encouraged to come up with a sampling of their own and compare it to the sample chosen using random numbers. Each student is asked to justify his/her sample based on bias/non- bias. The random numbers sample is also to be criticized since, when the population itself is small, the sample drawn by this method may not be suitable; this is also discussed.

Various sets of data are provided to the students and they are asked to suggest and justify which representation is most suitable to display the results

Students are asked to explain and critique the process of a survey or experiment, how that might have contributed to or influenced the results (e.g., reliability of sampling procedures, bias, missing or incorrect information) and describe misuses of statistical or numerical data

Have students make a budget for a specific task like building a wall or managing a barbeque fundraising event.

SAMPLE LESSON #1

Data Collection

Overview

This lesson demonstrates the inclusion of the Essential Learning Outcomes in classroom teaching.

Specific Learning Outcomes

Students will:

1. Distinguish between different types of variables, qualitative and quantitative and discrete and continuous.

Resources:

- Video/DVD player with screen and speakers.
- Video on the environment showing pollution especially the activities surrounding the Beetham or Forres Park garbage dump.
- Flip chart/white/black board.

Set Induction

Students are shown a 3-minute video reflecting the pile up of garbage. It reflects what goes on at the Beetham dump. The impact on living conditions on the neighbouring communities is showcased as well as on the rivers and Gulf of Paria. The demise of fish and dolphins in the Gulf is highlighted because of plastic bottles reaching the seas through pollution of rivers.

Strategies

- Problem Posing
A problem is posed about the impact of environmental pollution on their life.
- Cooperative learning
Students are organized into groups of 5. Discussions are guided onto how they felt and what actions can be taken e.g. create an awareness campaign on environmental issues/ initiate recycling projects in their school.

Each group identifies different issues related to garbage disposal e.g. types of garbage, names/ location of garbage dumps has to collect data on a particular topic, rate of decomposition of different materials like diapers, tins, plastics, cardboard boxes, death rate of dolphins, various environmental concerns. A list of variables is presented from the entire class.

- Whole group discussion
Teacher groups the variables into 2 categories - qualitative vs. quantitative and then continuous and discrete. Students continue to add to the list. Students explain why a particular variable belongs to a certain category.
- Regroup
Students identify an area of concern e.g. dump sites, biodegradable materials, impact on the seas, impact on the rivers, air pollution etc and create a subset of variables related to their activity.

Assessment

Students use a generated list of variables to distinguish among variables using a given bubble chart.

STANDARDS

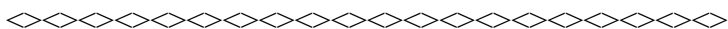
MODULE 2

Mathematics Standards

MODULE 2: USING THE EVIDENCE

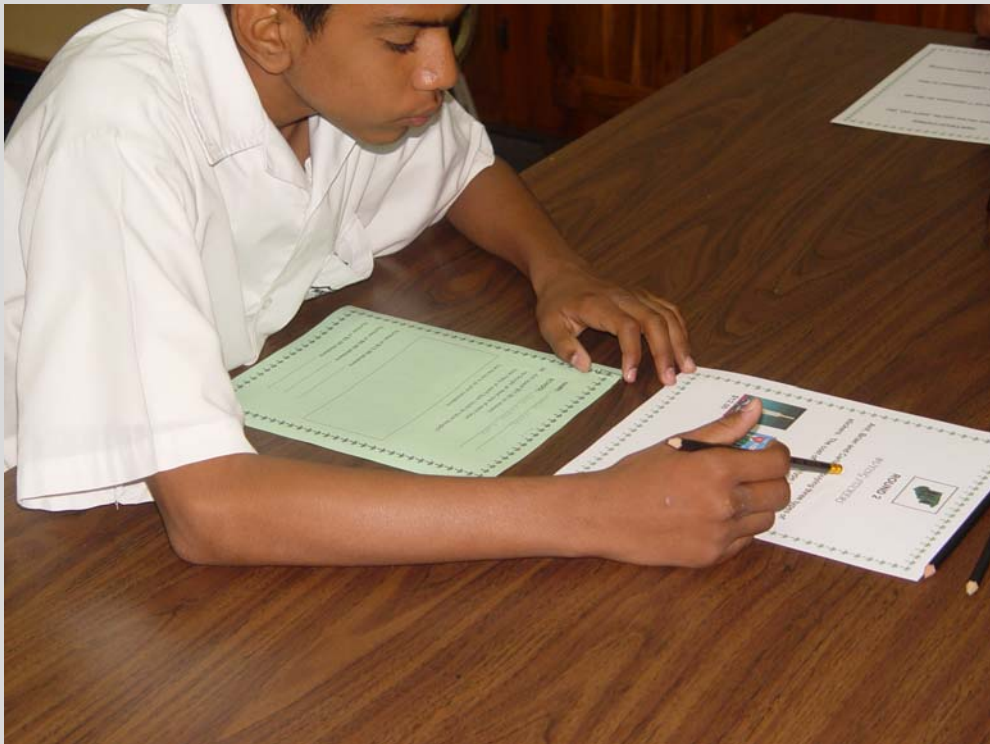
At the end of Form 5, students will:

- 1. Understand how to collect, use, present and represent data formally and correctly using technical terminology and without violating basic ethical principles.**
 - 1.1 Apply skills of tallying, tabulating, graphing and presenting data as evidence to support basic assertions;
 - 1.2 Apply basic principles to preserve confidentiality and integrity in collecting and presenting data;
 - 1.3 Recognise the type of data that could be best represented using different Bar Charts, Histograms, Frequency Polygons, cumulative frequency curves;
 - 1.4 Use a basic technological device as a tool in data analysis;
 - 1.5 Use a modern, basic, technological tool in the collection and in the analysis of data
 - 1.6 Correctly identify primary and secondary sources of data;
 - 1.7 Correctly describe ways of drawing samples;
 - 1.8 Distinguish between reputable and non-reputable sources of information on the World Wide Web;
 - 1.9 Use suitable evidence to evaluate claims and conclusions made in reports, articles, news items, studies and statements made by individuals for validity and bias.
- 2. Students will apply basic statistical reasoning and statistical tools to make informed judgements about simple everyday occurrences in the environment.**
 - 2.1 Simulate simple statistical experiments using a computer;
 - 2.2 Make and test simple hypotheses using basic theoretical ideas and data;
 - 2.3 Perform basic computations of theoretical probability of winning or losing familiar games of chance;
 - 2.4 Use probability to explain and evaluate the risk involved in making simple decisions in real life situations.



MATHEMATICS

MODULE 3



MODELLING WITH MATHEMATICS

MODULE 3: MODELLING WITH MATHEMATICS

This module includes:

Rationale	2–M3–1
Aim	2–M3–1
Goals	2–M3–1

Unit 1 Visualizing algebra

General Outcomes	2–M3–2
Specific Outcomes by Levels	
- Core	2–M3–3
- Extensions	2–M3–4
Content	2–M3–4
Instructional Strategies/Methods	2–M3–5
Suggestions for Assessment	2–M3–5
Resources	2–M3–5
Suggested Teaching and Learning Activities	2–M3–6

Unit 2 Understanding algebraic equations

General Outcomes	2–M3–7
Specific Outcomes by Levels	2–M3–7
- Core	2–M3–7
- Extensions	2–M3–8
Content	2–M3–9
Instructional Strategies/Methods	2–M3–9
Suggestions for Assessment	2–M3–9
Resources	2–M3–9
Suggested Teaching and Learning Activities	2–M3–10

Unit 3 Using algebra in problem-solving

General Outcomes	2–M3–11
Specific Outcomes by Levels	2–M3–11
- Core	2–M3–11
- Extensions	2–M3–12
Content	2–M3–12
Instructional Strategies/Methods	2–M3–12
Suggestions for Assessment	2–M3–13
Resources	2–M3–13
Suggested Teaching and Learning Activities	2–M3–14
Sample Lesson	2–M1–16
Standards	2–M3–27

MODELLING WITH MATHEMATICS

Rationale

The use of mathematical models to understand the world is one of the appealing aspects of Mathematics as a discipline. Problem-solving can be made easier by using symbolic representation to view simplified versions of real world situations. This could then enable a much more comprehensive exploration of complex real-world situations.

Aim

This module is designed to develop aspects of technological competence, aesthetic expression, communication, problem - solving and personal development through the use of Algebra, Relations, Functions, Co-ordinate Geometry, Graphs and Inequalities.

It is designed to help the student to analyse and solve problems through the use of technology, graphs, charts and algebraic thinking. It also seeks to develop advanced problem- solving skills and provides the opportunity to apply these skills to real life situations, in local and global contexts. Additionally, it provides opportunities for students to develop understanding and appreciation for the role of mathematics in important aspect of life situations - social, economic and political.

Goals

This module will support the attainment of outcomes in which students:

1. enhance problem-solving skills through the use of algebraic thinking and the use of models (AE, C, PD, PS, TC);
2. enhance critical thinking and enquiry skills (CIT, PS, PD);
3. develop appreciation of the precise nature of Mathematics (PD);
4. stimulate intellectual curiosity (PD, C, PS, TC);
5. enhance evaluating skills (C, CIT, PD);
6. enhance technological skills (AE, C, CIT, TC);
7. develop appreciation of the use of patterns in expressing mathematical ideas (AE, C, PS);
8. develop communication skills in the use of charts, graphs and symbols (AE, C);
9. enhance spatial reasoning skills in analyzing patterns and relationships of points in space (AE, C, PD, PS, TC);

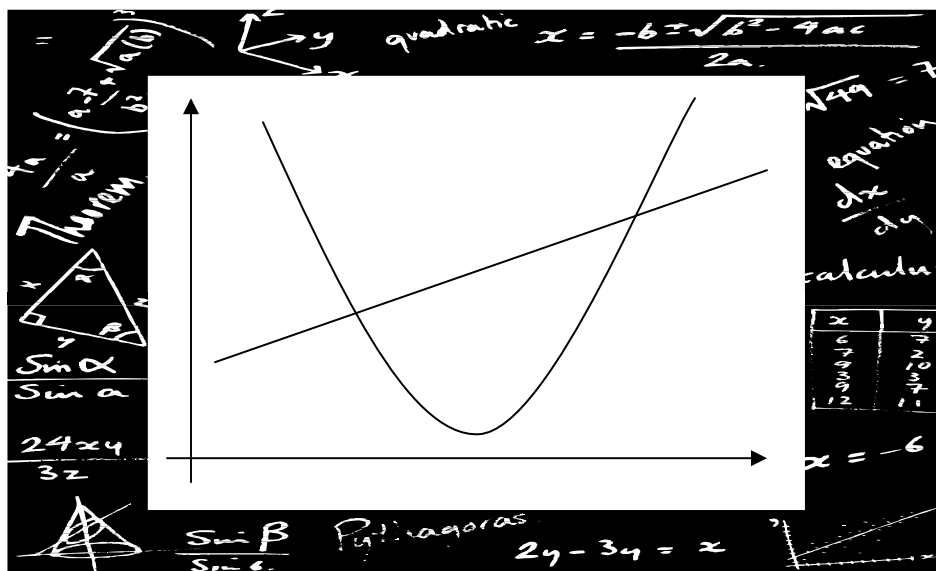
This module comprises three units:

Unit 1 – Visualizing Algebra

Unit 2 – Understanding Algebraic equations

Unit 3 – Using Algebra in problem solving

UNIT: 1



UNIT 1 – VISUALIZING ALGEBRA

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding of methods to analyze, share and evaluate information expressed using symbols, graphs, tables and other visual displays (AE, C, PS, TC);
2. demonstrate understanding of the relationships expressed by given linear and quadratic equations (C, PD, PS);
3. manipulate and display linear and quadratic graphs using technological tools (TC, C, AE);
4. demonstrate basic appreciation of formal mathematical enquiry (CIT, PD);
5. solve problems through the use of models (C, CIT, PS);
6. demonstrate appreciation for the use of mathematical symbols and models making generalizations (C, PS).

SPECIFIC OUTCOMES

CORE

Students are expected to:

1. state the characteristics that define a function and represent functions using function notation, for example, $f(x) = 2x + 5$, $f: x \rightarrow 2x + 5$, for a given domain;
2. determine function values for any domain of a linear or quadratic function;
3. determine whether a relation is a function by examining ordered pairs, arrow diagrams and graphs;
4. use functional notation to express (up to combinations of functions);
5. determine domain and range values of a function or combination of two (2) functions;
6. make connections between a function and its inverse and use arrow diagrams, graphs or functional notation to express a function or its inverse;
7. use algebraic methods to determine inverse relations, $f^{-1}(x)$ where $f(x)$ is a function of type $f(x) = ax + b$ or $f(x) = ax^2$ and establish that some inverses are functions;
8. use calculators and/or computers to generate combinations of numbers that increase or decrease linearly;
9. use charts, graphs and tables to determine if there is a linear relationship between two variables;
10. use properties (gradient, intercept with axes) of straight lines on the Cartesian coordinate system to make statements about whether the lines are parallel, perpendicular or cutting at acute or obtuse angles;
11. generate linear graphs on a graphing calculator and or computer;
12. use a calculator, overhead projector, computer and/or other available technologies and presentation tools to show features of linear graphs;
13. determine whether a pair of values satisfy quadratic relations $x \rightarrow ax^2$, $x \rightarrow (x + b)^2$, $x \rightarrow ax^2 + bx + c$;
14. distinguish between linear and quadratic expressions;
15. determine the factors of a quadratic expressions of types $ax^2 + bx + c$ and $a^2x^2 - b^2y^2$, where a , b and c are integers;
16. distinguish between linear expressions and linear equations;
17. solve simple problems using quadratic equations;
18. make connections between quadratic expressions written in algebraic form and numbers such as 267 (connect $2x^2 + 6x + 7$ with 2 hundred and sixty seven).

EXTENSIONS

Students are expected to:

19. *generate quadratic graphs on a graphing calculator and/or computer;*
 20. *distinguish between the graphs of $f(x) = x$, $h(x) = x^2$, $g(x) = x^3$ and $F(x) = \sqrt{x}$ in the first quadrant;*
 21. *use a calculator, overhead projector, computer and/or other available technologies and presentation tools to show features of quadratic graphs;*
 22. *use a graphing calculator and/ or computer to show different orientations of linear and quadratic graphs;*
 23. *use a calculator, overhead projector, computer and/or other available technologies and presentation tools to show features of periodic graphs such as $f(x) = \sin x$, $f(x) = \cos x$ and $f(x) = \tan x$;*
 24. *make connections between a function and its inverse using graphs and establish a relationship between the two;*
 25. *use algebraic methods to determine inverse relations, where $f(x)$ is a function of type $f(x) = ax^2 + bx + c$, establish that $(fg)^{-1} = g^{-1}f^{-1}$ for given domains and use inverse functions to solve problems of type $f(x) = a$.*
-

Content

- ❖ Relations and functions
- ❖ Linear expressions
- ❖ Linear graphs
- ❖ Gradient (of lines)
- ❖ Linear equations
- ❖ Quadratic expressions
- ❖ Factorization of quadratic expressions
- ❖ Quadratic relations [$y = x^2$, $y = ax^2$, $y = ax^2 + bx + c$, $y = a(x + b)^2 + c$]
- ❖ Quadratic graphs
- ❖ Comparison of gradients
- ❖ Point/s of intersection
- ❖ Transformation of linear and quadratic shapes
- ❖ Periodic graphs
- ❖ Modelling

Instructional Strategies/Methods

- Investigations
- Computer-aided activity
- Use of manipulatives
- Simulations
- Demonstrations

Suggestions for Assessment

- ✓ Problem-solving tasks
- ✓ Examination of student journals
- ✓ Observation
- ✓ Worksheets
- ✓ Presentations

Resources

- Manipulatives
- Calculators (graphing)
- Computers and software
- Activity sheets
- Charts
- Presentation software and hardware
- Overhead projector
- Cameras
- Scanners
- Graphing software



SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

Investigating Graphs

1. Activities and investigations of graphs are used for achieving a number of the learning outcomes in this module. The experience could also be used to develop students' abilities to use models in solving problems as well as to learn other problem-solving strategies.

Students can learn quite a lot about linear and quadratic functions through trying to fit lines or quadratic curves to sets of given points on a graphing calculator. They can vary values of the parameters until an appropriate model is found passing through a number of points. For example, the teacher could have students mark off or highlight certain coordinates on the screen. Students could start with a particular line or curve and be guided to vary parameter values in the calculator that will effectively change position, shape and gradient of the display on the screen. Students could attempt to get the ideal shape to model the points highlighted. They can investigate why other points do not fall on the line or curve. They can use digital systems such as cameras or scanners to keep records of shapes and properties of different graphs.

Students should be able to discover why a pattern of points in a set may be linear. (For example, if there is a fixed ratio between differences of y and x coordinates for different pairs of points).

2. Communication skills could be enhanced in this Unit. The use and understanding of mathematical terms, for example, could be developed by having students, on a daily basis, keep a journal of new mathematical terms and their meanings. Examples of their use could be kept in a portfolio or in an electronic organizer. Each day, or after each lesson, terms and meanings could be added to a list and a rethinking based on new knowledge or reinforcement of old meaning could be done on those terms already on the list.
-

UNIT: 2



UNIT 2 – UNDERSTANDING ALGEBRAIC EQUATIONS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding of the use of symbolic and geometric forms to model linear and quadratic relationships (AE, C, PS);
2. demonstrate high level competence in manipulating basic symbolic forms (C, PD, PS);
3. demonstrate understanding of the characteristics of classes of linear and quadratic functions (PS, PD);
4. demonstrate more advanced algebraic reasoning in solving problems. (C, PS, PD).

SPECIFIC OUTCOMES

CORE

Students are expected to:

1. use reasoning skills, knowledge of number and equation to form simple equations of numbers e.g. $3 + 2(4 + 2) = \dots + 2(3 + 1)$;

2. solve linear equations in one variable including the types

- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $ax + b(cx + d) = ex + f(gx + h)$
- $\frac{a(bx + c)}{m} + \frac{d(ex + f)}{n} = \frac{g}{r}$,

where a, b, c, d, e, f and g are real numbers and m, n and r are integers;

3. solve worded problems involving linear equations;
4. model real life situations using linear equations;
5. analyze and appreciate the changes that occur in the graph of $ay = mx + c$ as a, m and c take on different values;
6. distinguish between quadratic expressions and quadratic equations;
7. perform the operations of addition, subtraction, multiplication by a factor and division by a number, on quadratic expressions and equations;
8. rearrange and reorganize linear and quadratic expressions to give alternative forms.

EXTENSIONS

Students are expected to:

9. *divide a quadratic expression by a linear factor;*
 10. *state the equation of the line of symmetry of a parabola given its (the parabola) equation;*
 11. *determine the coordinates of the roots and turning point of a parabola from its equation;*
 12. *draw the graph of a parabola given its equation;*
 13. *determine points of intersection of a straight line and a parabola by drawing, observation (in simple cases) and by calculation;*
 14. *solve problems involving quadratic equations with and without the quadratic formula;*
 15. *predict the maximum number of intersection points of a parabola with the x axis;*
 16. *use properties of point of intersection and symmetry to create and analyze simple patterns involving straight lines and parabolas;*
 17. *compare the rates of change of two variables where the variables are in a linear relationship (example velocity-time and distance-time).*
-

Content

- ❖ Distributive law
- ❖ Associative law
- ❖ Gradient (rate of change)
- ❖ Solution of linear and quadratic equations
- ❖ Completion of squares
- ❖ Quadratic formula
- ❖ Changing the subject of a formula

Instructional Strategies/Methods

- Collaborative learning
- Investigations
- Problem based strategies
- Simulations
- Computer-aided instruction
- Discussion
- Research projects

Suggestions for Assessment

- ✓ Journals
- ✓ Paper and pencil test
- ✓ Performance tasks
- ✓ Presentations
- ✓ Worksheets

Resources

- Manipulatives
- Computer and software
- Graphing calculators

Suggested Teaching and Learning Activities

In teaching this module, the following activities may be useful:

Cellular phone bills

Understanding of straight-line models can be developed through engagement of students in familiar everyday activity. Almost every student would have been exposed to activities using land and 'cell' phones. Telephone calls may cost 84 ¢ for the first 3 minutes and 30 ¢ per minute for each subsequent minute.

The teacher could generate activity in which students develop tables of values giving total cost for calls of different length. Total cost can be modelled using a formula:

$$\text{Total cost} = 84 \text{ ¢} + 30 \times \text{number of minutes}$$

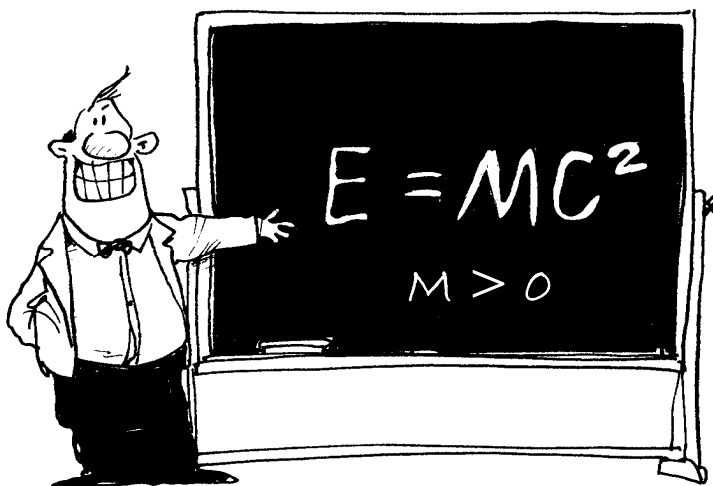
Students themselves should derive this model.

If 'cell' phones are permitted in the classroom, actually making calls on the cell phone and checking total cost could monitor cost. This is an excellent opportunity to expose students to the use of the technology properly and to examine ethical issues involved in its use.

It may also be possible to achieve the learning outcomes by simulations involving concrete material or the computer.

Students could draw graphs to model total cost as the length of calls varies. They can investigate changes in the graphs for different lengths of calls, cost per minute and cost for first three minutes.

UNIT: 3



UNIT 3 – USING ALGEBRA IN PROBLEM-SOLVING

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding of relationships expressed by linear and quadratic inequalities (AE, PS, C);
2. develop enquiry skills (CIT, PD, PS, TC);
3. demonstrate critical thinking skills (C, CIT, PD, PS).

SPECIFIC OUTCOMES

CORE

Students are expected to:

1. use knowledge of number and reasoning to write the symbols $<$, $>$, \leq and \geq correctly in numeric statements (e.g. $3 + 8 < 8 + 8$ or $5 + 4 \geq 5 + 3.4988$) ...;
2. write pairs of coordinates that satisfy the inequalities $y < ax + b$; $y \leq ax + b$; $y > ax + b$; $y \geq ax + b$;
3. write solution sets to linear inequalities using words symbols and graphs;
4. state largest and smallest values satisfying two linear inequalities in one variable;
5. solve problems involving linear inequalities in one variable;
6. distinguish between linear and quadratic inequalities.

EXTENSIONS

Students are expected to:

7. write possible solutions simultaneously satisfying two or more linear inequalities in two variables;
 8. determine regions represented by systems of inequalities in two variable;
 9. represent regions diagrammatically and on various display screens;
 10. determine whether a given set of points lie in a region;
 11. determine the solution set for a given quadratic inequality;
 12. use graphical displays to show points on the Cartesian plane that satisfy a given quadratic inequality;
 13. determine whether a given set of points satisfy a quadratic inequality;
 14. formulate and solve simple linear inequalities to model real life situations involving restrictions on quantities;
 15. create and solve problems, including real life ones, that involve inequalities (including the use of linear programming techniques).
-

Contents

- ❖ Linear inequalities
- ❖ Limits of inequalities
- ❖ Solution sets
- ❖ Number line representations
- ❖ Numeric and algebraic statements
- ❖ Regions defined by inequalities
- ❖ Quadratic inequalities
- ❖ Solutions of inequalities
- ❖ Modelling

Instructional Strategies/Methods

- Guided discovery
- Computer aided instruction
- Investigations
- Discussions
- Group work (These may be called investigative teams.)

Suggestions for Assessment

- ✓ Observation
- ✓ Performance tasks
- ✓ Group presentations
- ✓ Student journals

Resources

- Grids
- Graph sheets
- Computer and software
- Manipulative
- Presentation software and hardware



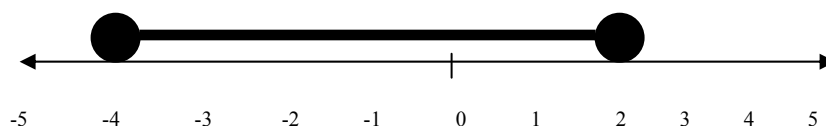
SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

1. Working with inequalities

1. It is suggested that the language of inequalities be developed at the same time that work on the concept is being done. For example, students need to read an inequality $2 \leq x < 5$ as all the values of x between 2 and 5 including 2 but not including 5.
2. Work on inequalities could also be done in conjunction with a review of number systems done in earlier work. Students could use number lines for example to describe sets of points using inequality notation. They can be encouraged to make comparisons between different representations and explore different ways in which they could represent individual sets as well as combinations of sets.

Example:



The set above may be described as:

$$\begin{aligned} & \{x : -5 < x \leq 2, x \text{ is an integer}\} \\ \text{or } & \{x : -4 \leq x \leq 2, x \text{ is an integer}\} \\ \text{or } & \{x : -5 < x < 3, x \text{ is an integer}\}, \text{ among others.} \end{aligned}$$

Students could be put in groups to explore different representations of a variety of inequalities and present their findings for the whole class to critique.

This is a simple exercise that can be used to encourage critical thinking and group participation.

2. Extension to inequalities in two variables

This will require the use of many more skills and deeper thought. In order to understand the relationship between points in the region $x + y < 8$ on the coordinate system, a student may be required to understand that the line $x + y = 8$ separates the Cartesian

space into the regions $x + y = 8$, $x + y > 8$ and $x + y < 8$. The teacher could separate the class into a number of groups and encourage them to investigate properties of points in the different regions produced by lines drawn on grids, graph paper or computer screens. They may also be given more than one line on the same diagram or display and be asked to investigate properties of the different regions.

Activities in this section should not be restricted to inequalities on the Cartesian system. Students should be encouraged to investigate inequalities in other situations as well.

These can include situations in which x and y represents cost or numbers of items produced.

SAMPLE LESSON #1

Unit 1:

Functions – Quadratic Graphs

Learning Outcomes

Students will:

- represent a function as a table, a graph and an equation
- determine the maximum value of a function
- compare quadratic functions.

Materials needed

- Activity sheet
- Graph sheet
- Graphing calculator

Instructional Plan

In the activity, students are given data in different forms in the context of a ball throwing contest. The data from Team A are shown in a table, the data from Team B are given by an equation and the data from Team C are shown on a graph.

- Have students read the first 2 paragraphs on the Activity Sheet and discuss what they observe about the height of the ball as the distance from the starting line increases. If the data points are plotted on a graph and connected, ask what shape do they think the graph would make?

(Students should observe that the height increases, then decreases. The shape is a parabola.)

- Have students read the third paragraph. Ask the class to describe the shape the equation would make, if a graph is drawn.

(Students should recognize that they have a quadratic equation which results in a parabola graph. The negative before the term in x^2 means that the graph opens down and has a maximum value.)

- Have students read the fourth paragraph. Ask the class to discuss what they can tell about the height the ball attained in relation to the distance from the starting line, by looking at the graph.
- After all discussions of the starting lines, the heights and the distances from the starting line for the 3 teams, ask students to determine which team they think won the contest and why.

Put students in groups of 3 or 4 to work on pages 2 and 3 of the Activity Sheet. They will need a graphing calculator to determine the equations for Team A and Team C.

Be sure to circulate around the class to help students with the effective use of the calculator.

Expected Results

Team A – Equation $y = -x^2 + 10x - 9$

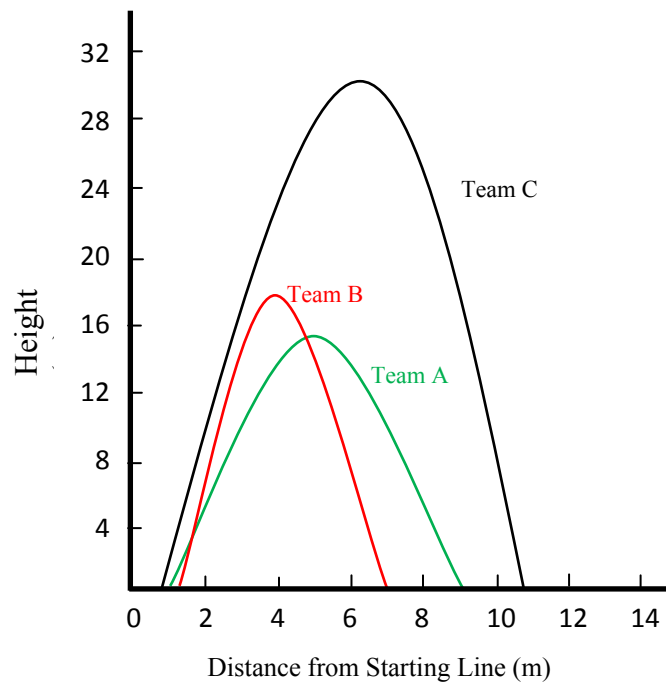
Team B – students may choose different points for their table.

x	1	2	3	4	5	7
y	3	12	17	18	15	-3

Team C - students may choose different points for their table.

x	1	2	4	6	8	10
y	3	12	24	30	24	12

- Equation - $y = -x^2 + 12x - 8$



ACTIVITY SHEET

Throwing a Ball Contest

Mr. King's class is holding a ball throwing contest on the football field. Teams of students have built structures that will launch a ball down the field. Mrs. Queen's class will judge the contest. They have various rules about how to measure each throw and how to determine which team wins.

Team A: This team used their structure to throw balls down the field. Students used motion detectors to collect data while the ball was in the air. They came up with the following table of data.

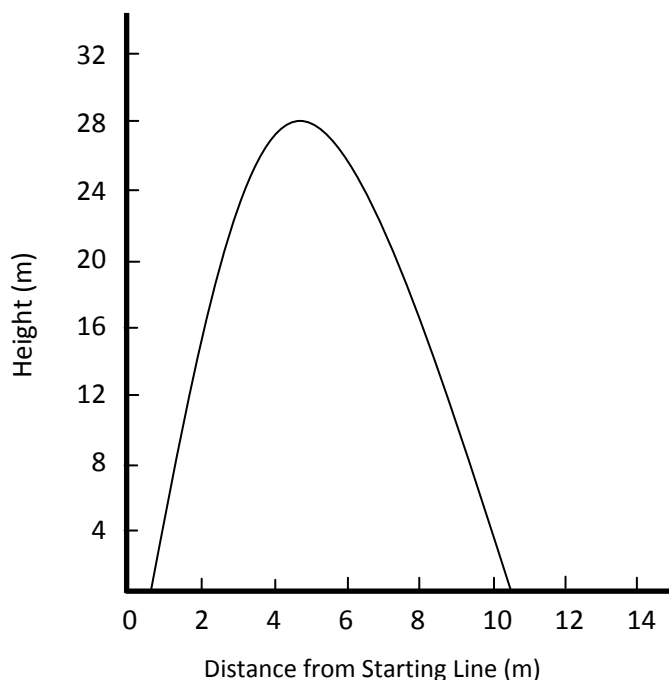
Distance from starting line (m)	2	4	5	7	9
Height (m)	7	15	16	12	0

Team B: This team conducted their ball throwing events and the class tracking the ball determined that the equation representing the path of the ball is:

$$y = -2x^2 + 15x - 10.$$

The x value is the distance from the starting line and the y value is the height of the ball from the ground. Both measures are in metres.

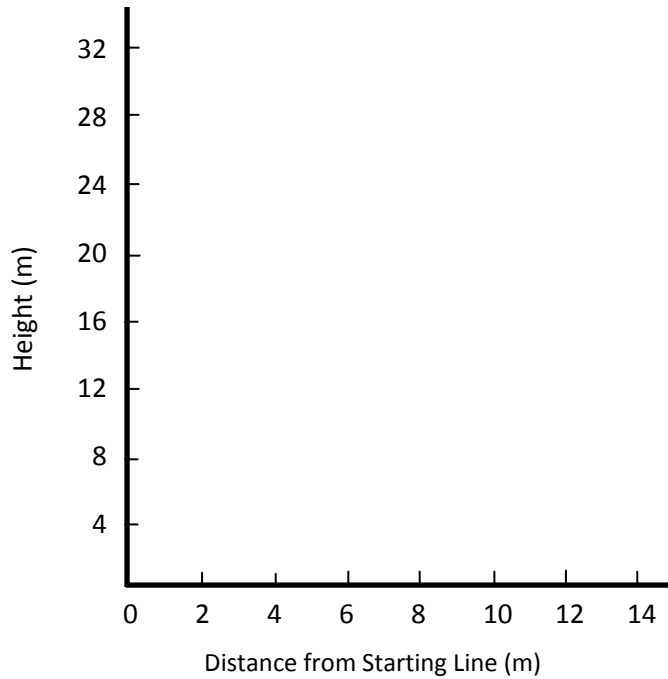
Team C: Like the other teams, this team engaged in throwing their balls. Some of the students from Mrs. Queen's class found the path of the ball as clearly shown on the graph below.



Question: Which team do you think won the contest? Why?

Team A

1. Using the data from Team A, determine an equation that describes the path of the ball.
Explain how you got your equation.
2. On the graph, draw a graph showing the path of Team A's ball.



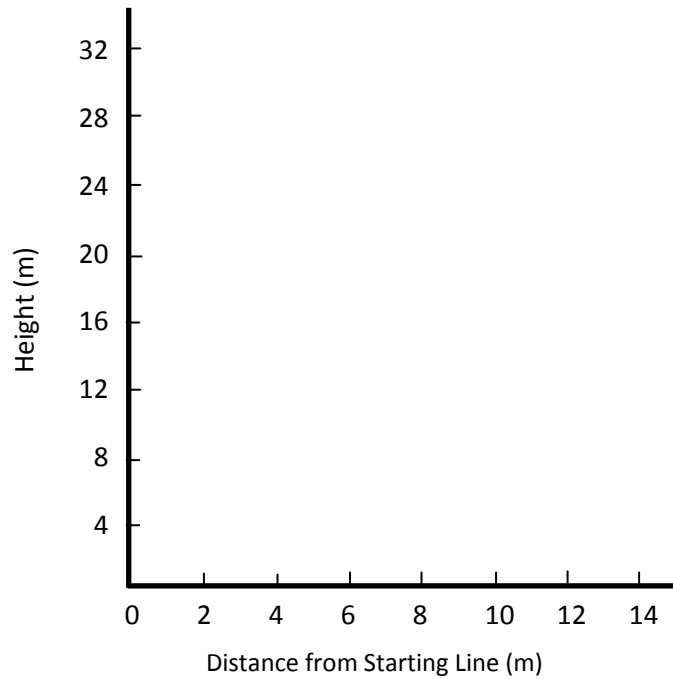
3. What is the maximum height that the ball reached?
4. How far was the ball thrown?

Team B

1. Using the equation from Team B, generate a table of values that shows different locations of the ball as it travelled through the air.

x									
y									

2. On the graph, draw a graph showing the path of Team B's ball.



3. What is the maximum height that the ball reached?
4. How far was the ball thrown?

Team C

1. Using the graph from Team C, generate a table of values that shows different locations of the ball as it travelled through the air.
2. What is the maximum height that the ball reached?
3. How far was the ball thrown?
-

SAMPLE LESSON #2

Mathematical Modelling

Learning Outcomes

Students will:

- Formulate and write a linear equation of the form $ax + by = c$;
- Present proof that the data being modelled is linear in nature;
- Make clear statements about data being modelled with equation;
- See relevance in the use of equations in a real world situation;
- Solve problems confidently using different strategies.

Resources:

- Statistical Data on provided sheets
- Graph Paper
- Mathematical Instruments
- Newspaper Advertisements

LEVELS OF LEARNING	LEARNING MODE	ESSENTIAL LEARNING OUTCOMES
- Comprehension - Application - Analysis	- Multiple Intelligences - Logical Mathematical - Visual/Spatial	- Problem-Solving - Communication

Set Induction

An advertisement from a daily newspaper on oil change and its value to automobiles is distributed to the class.

TEACHER STRATEGIES	STUDENT ACTIVITIES
The teacher divides the class into groups and presents data in a table showing number of oil changes (x variable) and cost of engine repairs (y variable) in one year.	Students engage in discussions. They analyze the data and seek to determine if the data represents a linear relationship.
The teacher instructs pupils to devise ways to determine if the data represents a Linear relationship and prove it.	They seek to plot the coordinates on a graph paper and draw a line of best fit. Further, they seek to derive the equation of the line by use of the intercepts and a calculated gradient.

TEACHER STRATEGIES						STUDENT ACTIVITIES						
No. of oil changes per year	0	1	2	3	3	4	4	5	6	7	10	
Cost of engine repairs \$	600	700	650	450	300	400	250	300	100	150	0	
<p>Groups are asked to use their derived linear relationship to predict the cost of engine repairs to a car that had 8 oil changes. (8 changes not being shown in the data)</p> <p>The teacher questions students on the practical value of the exercise.</p>						<p>Students use their graph reading skills or use algebraic substitutions to determine the cost of engine repairs to a car that had 8 oil changes.</p> <p>Students seek to explain the relevance of using linear modelling in such a real world situation. They also give reasons why such an approach should be used by persons in the automobile repair industry .</p>						

Assessment:

Students will be assessed on the following:

- Drawing a line of best fit
- Deriving a linear equation that matches their line of best fit
- Correctly using the linear model to make the required predictions
- Communicating the value of linear modelling.

Adapted from NCTM: Illuminations

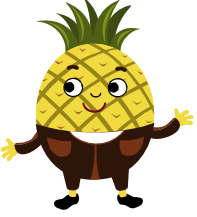

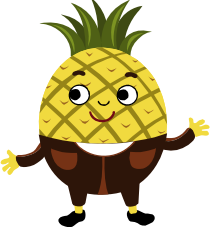


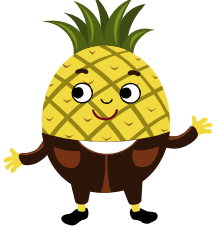
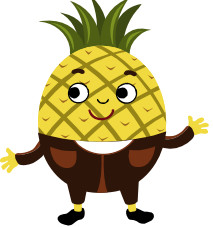
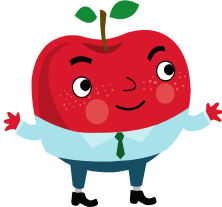




ALGEBRAIC THINKING: COMPUTER GAMES

A computer game, Fruit People, generates three different kinds of fruits.

Each fruit carries a different number of points.

The values on the extreme right represent the total number of points of all the fruits in each row.

Calculate the number of points for each fruit.


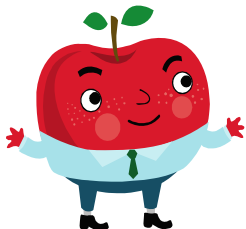
				66
				54
				40

ALGEBRAIC THINKING: COMPUTER GAMES

STUDENT ANSWER SHEET

Name: _____

School: _____

FRUIT	POINT VALUE
	
	
	

Time taken : _____

ALGEBRAIC THINKING: COMPUTER GAMES

TESTER'S INSTRUCTIONS




Read these instructions before presenting the visual.

1. This task requires you to solve a problem using information given on a visual.
2. You have a maximum of 8 minutes to solve it, but if you finish before, you are entitled to bonus marks, provided that the solution is correct.
3. Study the visual carefully and work out the solution. It is not necessary to show working but you may use the scrap paper to write, if you wish.
4. You will be given a recording sheet to write your answers.
5. When you have finished, say **STOP**, and the timer will stop the stop-watch.

Present the visual and start timing.

When the student says stop, stop the timer and record the time – let the student witness the recording.

ALGEBRAIC THINKING - ANSWER SHEET

FRUIT	POINT VALUE
	17
	15
	5

ALGEBRAIC THINKING - SCORING RUBRIC

	4	3	2	1	0	Score
ACCURACY		Fully correct- all three values correct	Partially correct- two values correct	Partially correct- one value correct	Not correct- no values correct	
TIME		Completed in less than 3 minutes	Completed in more than 3 but less than 4 minutes	Completed in more than 4 but less than 5 minutes	Completed in more than 5 minutes	
STRATEGY	Appropriate and efficient, deducing 1 or + 1 ap = 20 and solving accordingly	Setting up all 3 equations and attempting to solve pairs of equations	Setting up equations with at least one correct	Not efficient - use of trial and error without spotting relationships, making random guesses	Inappropriate strategy, will not lead to a solution.	
TOTAL						

STANDARDS

MODULE 3

Mathematics Standards

MODULE 3: MODELLING WITH MATHEMATICS

At the end of Form 5 will be able to:

1. Understand the concept of a function and use function notation to represent functions, its inverse and composition of functions.

- 1.1 Describe the concept of a function, use function notation and determine whether a given relation is a function;
- 1.2 Determine the domain and range of a function;
- 1.3 Determine the inverse of a function and whether this inverse relation is a function;
- 1.4 Determine the composition of functions;
- 1.5 Solve problems involving functions and their inverses.

2. Create and describe relationships with numbers and graphs using technology.

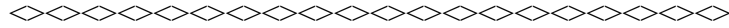
- 2.1 Investigate and understand graphical representations of linear and quadratic relations;
- 2.2 Analyze relationships between graphs when quantities change value or when quantities are equal in value;
- 2.3 Investigate and describe varying transformations of graphs.

3. Use algebraic methods to represent, analyze and solve mathematical and practical problems involving patterns and functional relationships.

- 3.1 Recognise, represent and distinguish between different types of expressions and equations;
- 3.2 Understand and represent real life situations in algebraic forms using linear and quadratic equations as is appropriate;
- 3.3 Solve linear and quadratic equations using appropriate method;
- 3.4 Demonstrate an understanding for algebraic expressions written in different forms.

4. Read, listen and discuss mathematical and real life situations; analyze and use the information to make algebraic connections involving inequalities and solve problems presented to them.

- 4.1 Use mathematical language, notation and symbols to represent problem situations and mathematical ideas involving inequalities;
- 4.2 Understand and solve linear and quadratic inequalities using algebraic and graphical methods;
- 4.3 Investigate, appreciate and understand the existence of a solution set and the limits thereof;
- 4.4 Develop and use critical thinking skills to analyze and solve real life situations.



MATHEMATICS

MODULE 4



REASONING WITH SPACE

MODULE 4: REASONING WITH SPACE

This module includes:

Rationale	2–M4–1
Aim	2–M4–1
Goals	2–M4–1

Unit 1 Locating points in space

General Outcomes	2–M4–2
Specific Outcomes by Levels	2–M4–2
- Core	2–M4–2
- Extensions	2–M4–3
Content	2–M4–3
Instructional Strategies/Methods	2–M4–4
Suggestions for Assessment	2–M4–4
Resources	2–M4–4
Suggested Teaching and Learning Activities	2–M4–5

Unit 2 Using geometry to understand space

General Outcomes	2–M4–6
Specific Outcomes by Levels	2–M4–6
- Core	2–M4–6
- Extensions	2–M4–7
Content	2–M4–7
Instructional Strategies/Methods	2–M4–8
Suggestions for Assessment	2–M4–8
Resources	2–M4–8
Suggested Teaching and Learning Activities	2–M4–9

Unit 3 Exploring motions in space through transformations

General Outcomes	2–M4–10
Specific Outcomes by Levels	2–M4–10
- Core	2–M4–10
- Extensions	2–M4–11
Content	2–M4–11
Instructional Strategies/Methods	2–M4–11
Suggestions for Assessment	2–M4–12
Resources	2–M4–12
Sample Lesson	2–M4–13
Standards	2–M4–17

REASONING WITH SPACE

Rationale

This module offers students the opportunity to enhance their understanding of geometrical concepts, logical thinking and spatial reasoning skills in two-dimensional and three-dimensional space. A deeper understanding of geometry is essential to interpreting, understanding, and appreciating our world that includes our art, festival costumes, landscape, architecture, and our use of limited space. This module also allows students to model and investigate the properties and relationships among geometrical objects and to explore motion in space by exploring the transformation of objects and matrices in space.

Aim

This module is designed to develop aspects of Problem-solving, Communication, Personal Development, and Technological Competence through the use of hands-on and authentic experiences in modelling and solving real world geometrical problems.

Goals

This module supports the attainment of outcomes in which students will:

1. enhance problem- solving skills through a deeper understanding of geometrical concepts and the development of geometrical thinking and spatial reasoning skills (CIT, PS, PD);
2. communicate ideas using geometrical and logical thinking.(AE, C, PD, TC);
3. demonstrate understanding of transformations in 2-D and 3-D space.(C, PD, PS);
4. appreciate the use of transformational geometry in explaining motion in pace. (C, PD, PS, TC);
5. extend geometrical concepts, spatial reasoning and creativity to appreciate, interpret and model designs in local and global environments. (CIT, PS, TC).

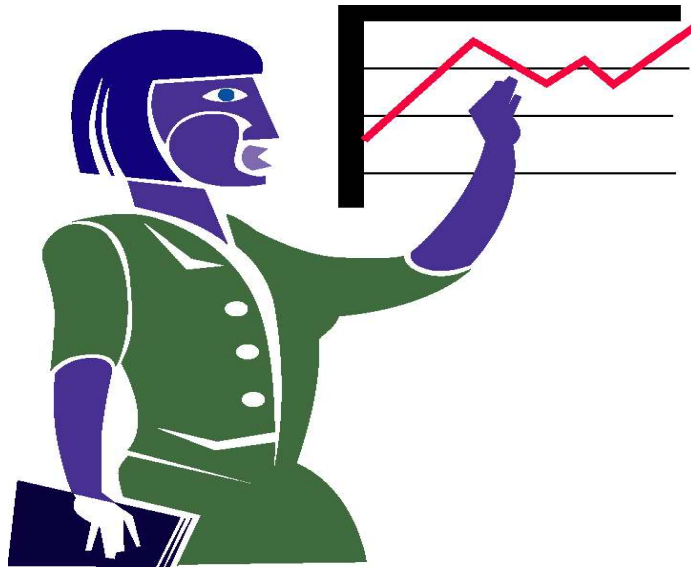
This module comprises three units:

Unit 1 – Locating Points in Space

Unit 2 – Using Geometry to understand space

Unit 3 - Exploring motion in space through transformations

UNIT 1



UNIT 1 – LOCATING POINTS IN SPACE

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding of space and the positioning of points (CIT, PD, PS);
 2. represent the coordinates of points, geometrical objects in the Cartesian plane (C, PS);
 3. apply coordinate geometry to represent straight lines in symbolic form (C, PD, PS);
 4. appreciate the usefulness of concepts in geometry to solve real-world problems (CIT, PD, PS).
-

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. locate the coordinates of points using co-ordinates on the Cartesian plane;
2. join sets of points located on the Cartesian plane to form straight lines and plan geometrical figure;
3. determine the gradient of a straight line using the coordinates of two points lying on the line;

4. compare the gradients of parallel and perpendicular lines;
 5. determine the midpoint and length (magnitude) of a straight line using the coordinates of two points lying on the straight line;
 6. determine the equation of a straight line using
 - the coordinates of two points lying on the straight line
 - one point and the gradient of the straight line/ its relationship to another straight line
 - the gradient of the straight line and its relationship to another straight line
 - the graph of the straight line;
 7. represent a straight line on Cartesian plane and determine the intercepts on the axes (graphical and algebraic methods);
 8. write simple mathematical statements to support or refute conjectures based on Coordinate Geometry concepts;
 9. solve geometrical problems using concepts and properties associated with points, straight lines and plane geometrical figures.
-

EXTENSIONS

Students are expected to:

10. *sketch straight lines using the gradient and the y-intercept;*
 11. *model and solve problems involving real-life situations (straight lines only) e.g. business applications, speed-time/displacement-time graphs;*
 12. *visualise three-dimensional shapes from different perspectives and identify their cross section.*
-

Content

- ❖ The straight line
 - Slope
 - Length (magnitude)
 - Midpoint
 - General equation of a straight line
 - Parallel lines
 - Perpendicular lines
- ❖ Intercept of a straight line graph (x-intercept and y-intercept).
- ❖ Basic properties of two-dimensional shapes.
- ❖ Modelling real-life situations (straight lines only).

Instructional Strategies/Methods

- Activate prior knowledge
- Teacher demonstration
- Explorations
- Interactive learning boards
- Cooperative groups
- Independent work
- Games

Suggestions for Assessment

- ✓ Teacher Observation Checklist – Formative
- ✓ Performance task – rubric
- ✓ Poster – checklist
- ✓ Simulations
- ✓ Peer observation
- ✓ Student self-assessment
- ✓ Worksheet- solutions
- ✓ Models

Resources

- Grid paper
- Geo - board
- Interactive software
- Graphing calculator
- Plane and coloured paper
- Glue
- Pair of scissors

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activities may be useful:

Visual demonstrations

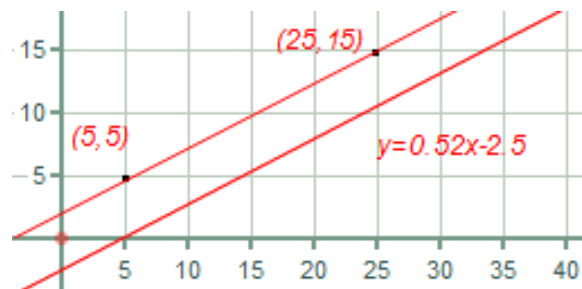
1. Exploring the gradients of parallel lines

Teachers will:

- guide students to plot a series of lines with equal gradients
- discuss patterns/properties observed.

Students will:

- plot a series of lines with equal gradients
- discuss patterns/properties observed
- Make journal entries on their findings.



2. Pattern Sequences

Teachers will:

- guide students to plot a basic geometrical shape as a motif for generating a pattern
- discuss patterns and how they may be developed

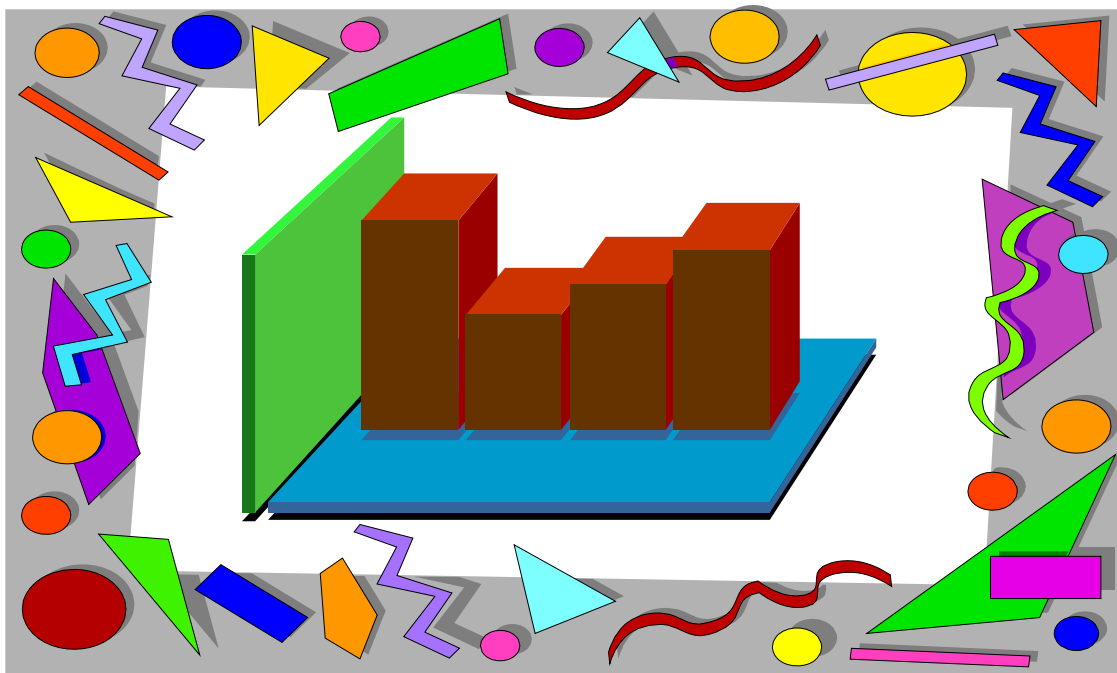
OR

- provide a demonstration on how a motif can be repeated to generate patterns.

Students will:

- plot a basic geometrical shape or a created shape as a motif
 - create pattern(s) by repeating one motif or a combination of motifs.
-

UNIT: 2



UNIT 2 - USING GEOMETRY TO UNDERSTAND SPACE

GENERAL OUTCOMES

The students will:

1. apply the properties and relationships among geometrical objects to solve problems (CIT, PD, PS);
2. appreciate the use of geometric models and trigonometrical methods in representing and solving simple real world problems (CIT, PD, PS).

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. solve geometric problems based on properties of straight lines and angle properties;
2. solve geometrical problems based on properties of
 - parallel and intersecting lines
 - plane shapes (triangles, rectangles, quadrilaterals, polygons and circles) including

- angle properties.
 - congruent triangles
 - similar figures
 - circles
 - solids in three-dimensions (prisms, pyramids, cylinders and spheres);
4. solve geometrical problems using Pythagoras' theorem;
 5. use given sides of right-angled triangles to determine values of the three basic trigonometrical ratios;
 6. solve right-angled triangles involving the three basic trigonometrical ratios, angles of elevation and angles of depression;
 7. apply three basic trigonometrical ratios to solve problems in two-dimensional real-life geometrical situations involving right-angled triangles such as design technology, architecture, landscaping;
 8. locate the positions of points in the Cartesian plane given its bearing from a point of reference;
 9. use sketches, maps, scale drawings or computer aided diagrams to explore and solve simple problems involving bearings;
 10. write simple mathematical statements to support or refute conjectures based on geometrical concepts.

EXTENSIONS

Students are expected to:

11. *solve geometrical problems, including three-dimensional real world problems using the sine and cosine rules, bearings and other geometrical concepts;*
12. *solve problems involving circle theorems associated with tangents and chord;*
13. *calculate the size of interior angles in polygons.*

Content

- ❖ Types of angles – acute, obtuse, reflex, right, straight, complementary, supplementary.
- ❖ Angles and straight lines – Vertically opposite, Adjacent, Angle at a point,
- ❖ Parallel lines and transversals - Corresponding, Alternate (Z angle), Co-interior
- ❖ Types of triangles – Equilateral, isosceles, scalene, right-angled
- ❖ Quadrilaterals – Square, Rectangle, Rhombuses, Parallelograms, Trapezium
- ❖ Polygons to include regular and irregular pentagons and hexagons
- ❖ Solid shapes – face, edge, vertex, height, base, surface area, volume

- ❖ Congruency and Similarity – conditions
- ❖ Pythagoras' Theorem
- ❖ Trigonometrical ratios – sine, cosine, tangents
- ❖ Sine rule, cosine rule
- ❖ Bearing, Angle of elevation, Angle of depression
- ❖ Scale drawing
- ❖ Circle properties – angles subtended on same chord/arc, cyclic quadrilateral etc.
- ❖ Circle Theorems – tangents etc.

Instructional Strategies/Methods

- Cooperative learning
- Computer graphics
- Demonstrations
- Guided instructions
- Investigations
- Research (using the Library, Internet, etc)
- Portfolios
- Visuals of Arab geometric art and Escher diagrams

Suggestions for Assessment

- ✓ Performance tasks
- ✓ Worksheets
- ✓ Questioning
- ✓ Problem-solving tasks
- ✓ Group projects – construction of figures/ models

Resources

- Books on art, geometrical patterns and designs
- Computer and Computer software
- Interactive board
- Activity sheets using Manipulative - plane shapes - sets of wooden, plastic or foam
- Solid shapes including transparent sets
- The World Wide Web or Libraries

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In teaching this module, the following activity may be useful:

1. Making and proving simple conjectures

Teacher will:

- *Discuss conjectures with students*
- *Use an example to help students develop understanding of how to prove conjectures.*

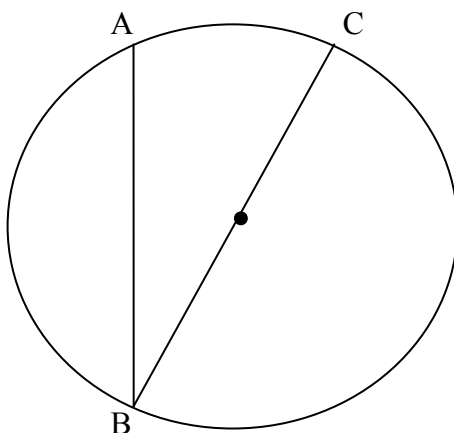
Students will:

- *Participate in class discussion by asking questions and make conjectures.*
- *Engage in trying to prove their conjectures.*

Example:

A chord is any straight line that meets a circle at two points.

Conjecture: The length of the chord AB is less than the length of the diameter BC.



Proof:

Construction: Join the points A and C using a straight line.

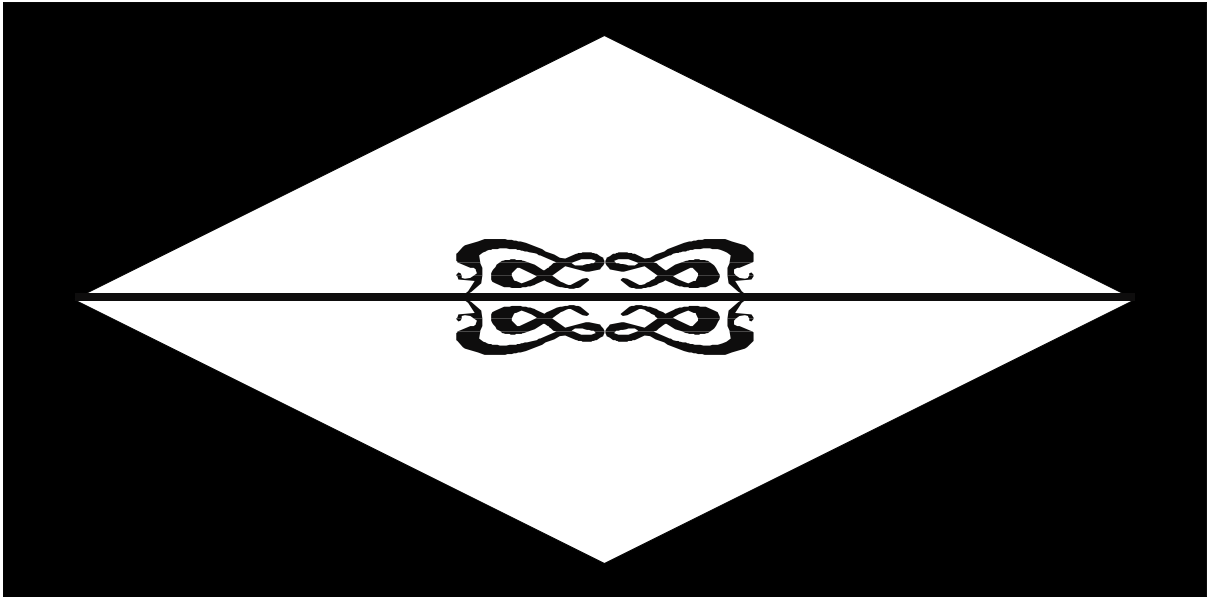
. BAC is 90° (A diameter subtends a right angle at the circumference)

\Rightarrow In the right-angled triangle ABC, BC is the longest side (The hypotenuse is the longest side in a right-angled triangle.)

\Rightarrow In triangle ABC, length of side BC is greater than length of side AB.

\Rightarrow Length of Chord AB is less than length of diameter BC.

UNIT 3



UNIT 3: EXPLORING MOTION IN SPACE THROUGH TRANSFORMATIONS

GENERAL OUTCOMES

Students are expected to:

1. apply and describe transformations of geometrical objects in the Cartesian plane (C, CIT, PD, PS);
2. represent linear transformations in the Cartesian plan using matrices. (C, PD, PS).

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. demonstrate understanding of Transformation, Translation, Reflection, Rotation, Enlargement, Reduction, and explore real-life application of each;
2. represent the images of transformations of a set of points, including straight lines, regular and irregular plane geometrical shapes in the Cartesian plane through
 - Translation using a Translation Vector
 - Reflection in a mirror line or in the Origin
 - Rotation about a centre of rotation in a given direction
 - Enlargement or Reduction by a given scale factor;

3. identify and describe the type of transformation given its object and image;
 4. identify the types of symmetry present in plane geometrical shapes
 - lines of symmetry
 - rotational symmetry
 - order of symmetry.
-

EXTENSIONS

Students are expected to:

5. *determine centres of Rotation and Enlargement for a given transformation;*
 6. *represent the images of two or more combined transformations on a set of points, straight lines, regular and irregular plane geometrical objects;*
 7. *deduce the geometrical relationship between an object and its image after undergoing two or more combined transformations in the Cartesian plane;.*
 8. *represent '2 x 2' matrices resulting from specified linear transformations of Reflection, Rotation, Enlargement and Translation;*
 9. *represent combined linear matrix transformations by an equivalent single '2 x 2' matrix.*
-

Content

- ❖ Transformations
 - Reflections - Symmetry
 - Rotations
 - Enlargements
 - Translations
- ❖ Linear transformations involving matrices

Instructional Strategies

- Demonstrations
- Explorations
- Cooperative learning
- Performance tasks
- Project work
- Role playing
- Workstations : hands - on activities

Suggestions for Assessment

- ✓ Observations
- ✓ Paper and pencil tests
- ✓ Portfolios
- ✓ Presentations
- ✓ Problem based worksheets
- ✓ Student journals

Resources

- Manipulative – Geo Reflectors, Geo - boards and transparent geometrical objects
- Computer software
- Geometrical instruments

SAMPLE LESSON

Unit 1

Scales and Bearings using Maps

Learning Outcomes:

Students will be able to:

- research their local area map using ‘Google earth’ and obtain area map where school is located;
- locate the position of their school on local area map;
- create their own local map indicating clearly the position of their school;
- repeat procedure for location of other school of interest in local area;
- create and present map of school and locations of interest.

Materials:

- Computer and Internet access
- Printer
- Multi-media projector
- Protractors
- Rulers
- Pencils
- Drawing paper
- Activity sheet

Overview

As citizens to move around the globe for travel or to build better lives, the ability to develop essential skills of global living is taking on a greater significance for the 21st Century teacher. The availability of maps from ‘Google earth’ through technology enhances classroom instructions and provides an excellent means of developing a working knowledge of maps.

This lesson enhances technological competence and citizenship as students explore maps of local areas using ‘Google earth’ and make power point presentations of their findings.

Introduction

The teacher introduces the lesson by asking students how many of them have ever used the Internet. It’s likely that most have. To those who have not, technology support will be provided by the teacher. Explain that for this activity students are to use the Internet to download the free “Google earth” program from the following website: earth.google.com/download.earth.html

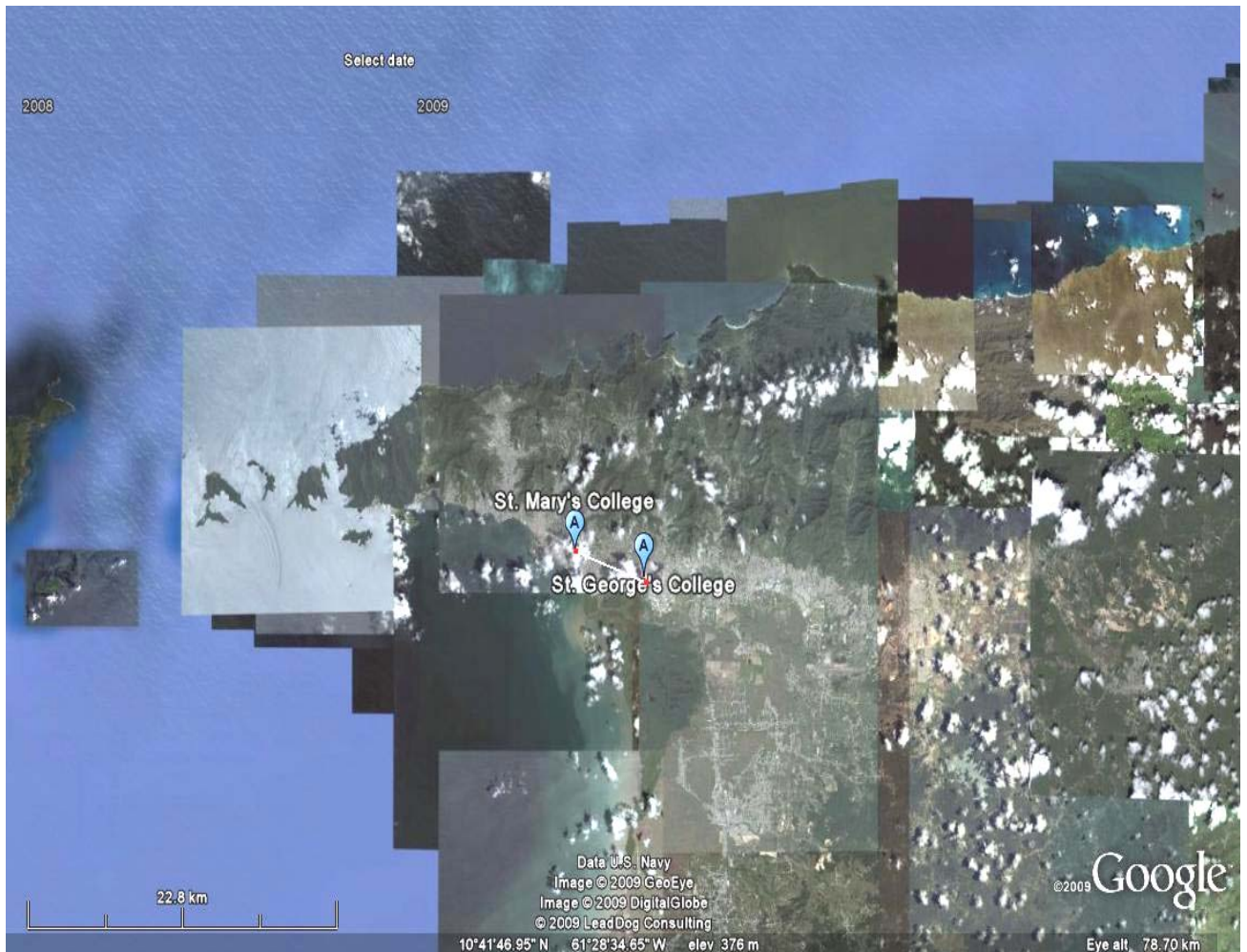
TEACHER ACTIVITY	STUDENT ACTIVITY
<ul style="list-style-type: none"> Teacher reviews use of: <ul style="list-style-type: none"> - scales on maps to determine distances using multi-media projector - protractor to measure angle using a map 	<ul style="list-style-type: none"> Students make brief notes.
<ul style="list-style-type: none"> The teacher distributes Activity Sheet and give guidelines. 	<ul style="list-style-type: none"> Students form flexible pairs.
<ul style="list-style-type: none"> The teacher facilitates Activity, makes observations and offers technological support where needed 	<ul style="list-style-type: none"> Students identify their reference point for the Activity Locate and record position and distance of school from reference point. Select other points of interest and record their positions and distances from references. Create map for assessment including computer aided map.
<ul style="list-style-type: none"> The teacher ensures Internet access available 	<ul style="list-style-type: none"> Students research information (legend) on local community as part of assessment

Assessment Strategy: Scoring Rubric

Poster or Power point Presentation with accompanying assessment criteria:

- Details on Maps - location of all the schools and areas of interest chosen by each group, students will
 - label locations and distances
 - verify accuracy of co-ordinates and distances
 - display creativity and effective use of colours
- Research
 - examine relevance of information to local context
- Group Presentation and Teamwork
 - appropriate information selected
 - creativity
 - clarity of speech in delivery
 - ability to hold interest of audience

An example of a map from “Google earth”



STANDARDS

MODULE 4

Mathematics Standards

MODULE 4: REASONING WITH SPACE

At the end of Form 5, students will be able to:

- 1. Locate a set of points using co-ordinates on the Cartesian plane and describe spatial relationships using co-ordinate geometry and other forms of representations.**
 - 1.1 Represent a set of points with real number co-ordinates on the Cartesian plane;
 - 1.2 Identify spatial relationships among points, straight lines and two-dimensional geometrical objects;
 - 1.3 Represent spatial relationships among points, straight lines and two-dimensional geometrical objects using other mathematical representations;
 - 1.4 Make and investigate simple conjectures about points, straight lines and two-dimensional geometrical shapes represented using Cartesian co-ordinates;
 - 1.5 Write simple mathematical statements to support or refute a conjecture made.

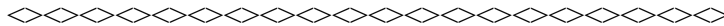
- 2. Investigate and analyse the characteristics and properties of two-dimensional and three-dimensional geometrical shapes and develop mathematical arguments about spatial relationships among them.**
 - 2.1 Describe physical properties and symmetries of two-dimensional and three-dimensional geometrical shapes.
 - 2.2 Explore spatial relationships among two-dimensional shapes including congruence and similarity.
 - 2.3 Make and prove conjectures about two-dimensional geometrical shape using geometrical ideas and spatial reasoning skills.
 - 2.4 Solve geometrical problems involving two-dimensional and three-dimensional shapes.
 - 2.5 Solve geometrical and real-world problems using trigonometrical relationships and bearings.

3. Model and solve problems using visualization, geometrical ideas and spatial reasoning.

- 3.1 Sketch and make accurate representations of two-dimensional and three-dimensional geometrical shapes using a variety of tools including technological tools;
- 3.2 Visualize three-dimensional shapes and identify their cross sections;
- 3.3 Model and solve geometrical problems using representations of two-dimensional and three-dimensional shapes;
- 3.4 Apply geometrical modelling to deepen understanding of geometry, other areas of Mathematics and other disciplines e.g. Art and Architecture.

4. Apply transformations and symmetries to analyse mathematical situations.

- 4.1 Represent single transformations of points, straight lines and two-dimensional shapes in the Cartesian plane using sketches, co-ordinates vectors and (2×2) matrices;
- 4.2 Describe single transformations of points, straight lines, two-dimensional shapes and (2×2) matrices;
- 4.3 Represent and describe combined transformations of points, straight lines, two-dimensional shapes and (2×2) matrices.



MATHEMATICS

MODULE 5



TAKING CONTROL OF SPACE

MODULE 5: TAKING CONTROL OF SPACE

This module includes:

Rationale	2–M5–1
Aim	2–M5–1
Goals	2–M5–1

Unit 1 Maintaining accurate measurements

General Outcomes	2–M5–2
Specific Outcomes by Levels	2–M5–3
- Core	2–M5–3
- Extensions	2–M5–3
Content	2–M5–4
Instructional Strategies/Methods	2–M5–4
Suggestions for Assessment	2–M5–4
Resources	2–M5–5
Suggested Teaching and Learning Activities	2–M5–6

Unit 2 Measuring shapes

General Outcomes	2–M5–8
Specific Outcomes by Levels	2–M5–9
- Core	2–M5–9
- Extensions	2–M5–9
Content	2–M5–10
Instructional Strategies/Methods	2–M5–10
Suggestions for Assessment	2–M5–10
Resources	2–M5–11
Suggested Teaching and Learning Activities	2–M5–12

Unit 3 Modelling shapes

General Outcomes	2–M5–14
Specific Outcomes by Levels	2–M5–15
- Core	2–M5–15
- Extensions	2–M5–15
Content	2–M5–16
Instructional Strategies/Methods	2–M5–16
Suggestions for Assessment	2–M5–16
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Sample Lesson	2–M5–17

Standards	2–M5–19
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TAKING CONTROL OF SPACE

Rationale

The ultimate goal of our modern society is to have command of our limited space so that we can lay the foundation for a safe, healthy and sustainable environment. This requires that the learner in the twenty-first century must develop skills that enable him or her to manoeuvre our limited space creatively and to be able to explore it sufficiently for a given purpose. Taking control of space will seek to deepen spatial awareness and understanding that can foster in learners aspirations of becoming active agents of change in both our local and global environments.

Aim

Module 5 is designed to develop aspects of aesthetic expression, citizenship, communication, personal development, problem - solving and technological competence using appropriate content in Measurement. It also seeks to enhance spatial reasoning and problem - solving skills, as well as practical knowledge in the use of measures, including those that are used in cultural contexts. In addition, this module will provide opportunities for the development of social responsibility and personal responsibility as learners master skills in measurements in a variety of real-world contexts.

Goals

This module supports the attainment of outcomes in which students will:

1. enhance estimation and computational skills (C, PD, PS);
2. gain an understanding of precision and use of measurement (PS, PC);
3. communicate information on measures using appropriate units and a given degree of accuracy (C, TC);
4. demonstrate improved self-awareness and understanding of the perceptions and feelings of others (PD);
5. express mathematical ideas in their common everyday language (C, CIT);
6. develop collaborative and decision-making skills and attitudes. (PD, PS).

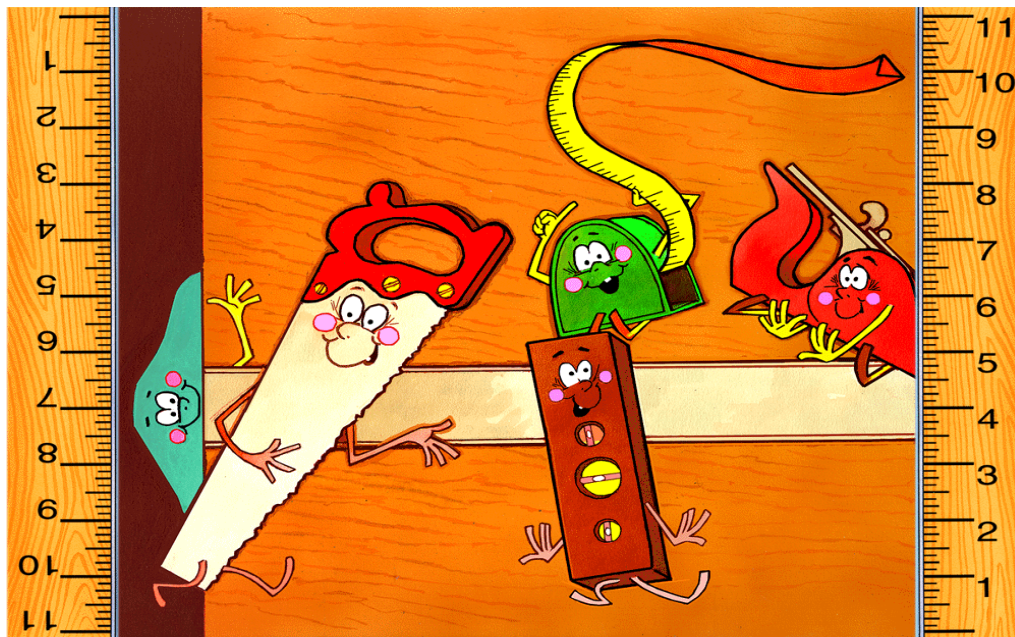
This module comprises two units:

Unit 1 — Maintaining Accurate Measurements

Unit 2 — Measuring Shapes

Unit 3 — Modelling Shapes

UNIT 1



UNIT 1— MAINTAINING ACCURATE MEASUREMENTS

GENERAL OUTCOMES

Students are expected to:

1. demonstrate an understanding of various measuring systems, including formal and informal ones (PD, PS, TC);
2. appreciate the need for precision, standardization and accuracy in making measurements (PD,PS);
3. demonstrate understanding of degree of accuracy(PD, PS, TC);
4. apply concepts of measurement and estimation appropriately (C, CIT, PS);
5. evaluate situations that require varying degrees of accuracy when making measurements (PD, PS);
6. develop co-operative and collaborative skills (C, CIT, PD);
7. apply logical reasoning in the selection of units of measurement (CIT, PD, PS);
8. demonstrate enhanced skills in the use of maps and models (C, PD, PS, TC).

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. use both the metric and imperial system of units to effectively compare large and small quantities in both systems of units, e.g., cm. with inch, kilometre with mile;
 2. distinguish between formal and informal measures;
 3. justify the use of a particular unit of measure for quoting a given quantity, e.g., use of yards instead of inches, of kilometres rather than metres in real life situations;
 4. measure quantities such as length, volume, temperature and angle to a given degree of accuracy;
 5. quote numerical results to designated degree of accuracy in a given situation;
 6. correctly convert quantities quoted in one set of units to other units , e.g., degrees to radians, millimetre to metre, and kilometre to mile;
 7. determine distances on maps and models in accordance with a given scale;
 8. create accurate drawings, charts and graphs given a particular scale requirement;
 9. use the computer and other technology to tell time in different countries and to compare with time in Trinidad and Tobago.
-

EXTENSIONS

Students are expected to:

10. *state and justify the degree of accuracy in any quantity when another related quantity is measured to a stated degree of accuracy. Only simple relations should be considered.;*
 11. *estimate the margin of error in measuring quantities;*
 12. *state the units of derived quantities obtained from addition, subtraction, multiplication and division of quantities in the same and/or different units*
(e.g., $\frac{km}{sec}$, $\frac{km/sec}{sec}$)
-

Content:

- ❖ Metric system of units
- ❖ Imperial system of units
- ❖ Degree of accuracy
- ❖ Errors
- ❖ Conversions
- ❖ Operations with units
- ❖ Specifications
- ❖ Scales
- ❖ Standard time
- ❖ Informal measures

Instructional Strategies/Methods

- Cooperative learning activity
- Demonstrations
- Discussion.
- Situation analysis
- Games
- Classroom practical learning centres
- Direct instruction
- Computer –based instruction
- Explorations
- Investigations
- Project work

Suggestions for Assessment

- ✓ Paper and pencil tests
- ✓ Performance tasks
- ✓ Projects
- ✓ Demonstration of understanding using manipulative and measuring tools
- ✓ Peer evaluation
- ✓ Presentations

Resources

- Calculators
- Charts
- Computer and software
- Internet facilities
- Maps and models
- Manipulative
- Measuring instruments
- Presentation software
- Presentation equipment

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In this module, the following activities may be useful:

1. Integrate Imperial and Metric Units across the Curriculum

At the Form 4/5 level, students should already have had a good grasp of the relative size of all the Metric units of measure and a working knowledge of many of the commonly used Imperial Units such as feet, mile and pounds. The teacher should use the opportunity to integrate use of both the Metric and the Imperial Units in different topics across the curriculum such as ratio and proportions and with different disciplines such as Science and Geography.

2. Review concepts of Ratio and Proportions

Revisit the concepts of ratios and proportions. Have students do activities in which they explore the results of mixing quantities in different proportions. For example, the resulting colour of a liquid produced by mixing 5cc of a liquid of one colour with 1 L of liquid of another colour.

3. Estimate distances on maps

Have students draw maps of regions such as parts of their school or village to given scales and use their maps to estimate distances between points in meters or kilometres. They may confirm their estimations from different sources such as architect plans and so on.

Students could also use the computer and Internet facilities to determine distances between different objects such as ships, aircraft in real time.

4. Measuring the speed of a cricket ball

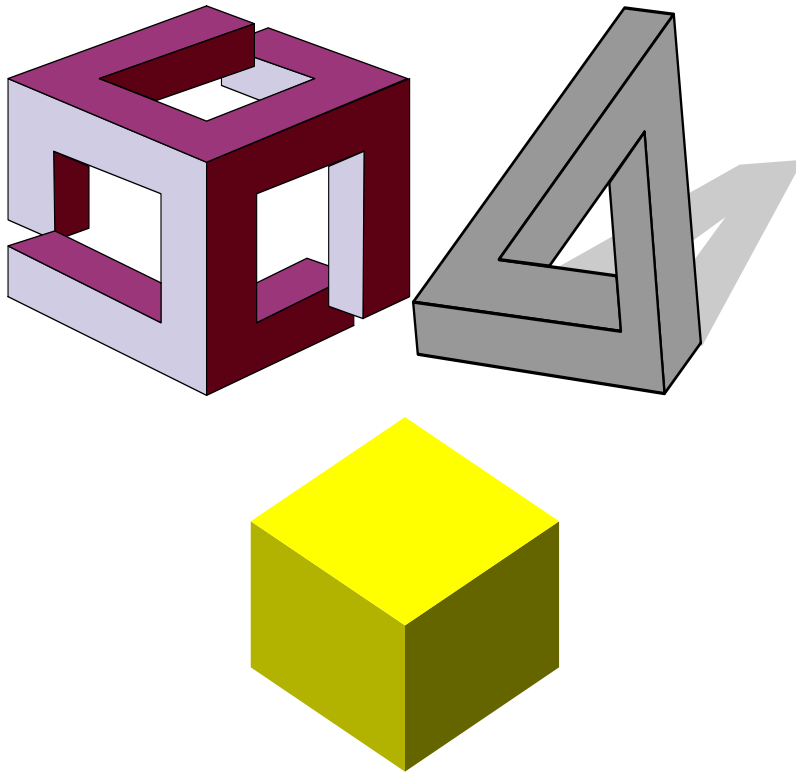
An example of an activity that can facilitate a number of outcomes in this Unit is one in which students take measurements of distances and speed of a cricket ball bowled by students indoors. Students can experiment by recording the heights at which the ball hits

a wall after dropping at different distances away from the wall. They can explore ways to make more accurate measurements and to determine the speed of the ball. Have them decide on the appropriate units to use for any measure, the instrument used and the level of precision required for a particular purpose. The teacher could arrange video taped sessions to record results and have students review their results.

5. Informal Measures

One way in which students could be made to appreciate the need for standard measures is to have them do various tasks using informal measures they confront in their daily encounters. For example, they often hear in a local context terms such as 'Arm's length', 'Whole half hour', 'Right down the road', 'Real big', 'Just now', being used to describe various quantities. Have students describe/interpret the meanings of these informal measurements and determine the problems with their use in different situations.

UNIT: 2



UNIT 2 — MEASURING SHAPES

GENERAL OUTCOMES

Students are expected to:

1. develop and use formulae for area and volume;
2. develop and use procedures for computing and estimating areas and volumes;
3. understand how to link knowledge of formulae and shape of figures to compute perimeters and areas and solve problems;
4. demonstrate an understanding of the Pythagorean relationship using models.

SPECIFIC OUTCOMES

CORE

Students are expected to:

1. estimate volumes of common solids such as cuboids and cylinders;
2. determine unknown quantities in standard formulae for perimeter and areas of triangles, trapezia, parallelograms and circles;
3. solve problems involving perimeter and area of triangles, rectangles and circles (including the use of trigonometric formulae);
4. determine the change in area of a rectangle if the length or breadth changes by a given amount or by a percentage;
5. determine the change in either the length or breadth, as one remains constant while the area changes by a given amount;
6. create and use formulae to compute perimeter and area of polygons, arcs, and sectors;
7. compute perimeters of compound shapes;
8. create and use formulae to compute surface area and volume of cubes, cuboids and cylinders;
9. estimate surface areas and volumes of cylinders;
10. compute areas of compound shapes formed with rectangles and circles;
11. measure and compute surface area and volume of cuboids;
12. measure and compute surface area and volume of solids composed of two cylinders;
13. compute total length of the edges of cuboids;
14. apply the Pythagorean relationship in problem - solving and real- life situations.

EXTENSIONS

Students are expected to:

15. *create and use formulae to compute surface area and volume of cones, spheres and right prism;*
 16. *estimate surface areas and volumes of right prisms;*
 17. *compute areas of compound shapes formed with sectors;*
 18. *measure and compute surface area and volume of solids composed of Pyramids.*
-

Content

- ❖ Formulae
- ❖ Areas of polygons
- ❖ Surface area of cubes, cuboids, cylinders, cones, spheres and pyramids
- ❖ Volume of cubes, cuboids, cylinders, cones, spheres and cylinders
- ❖ Area computation
- ❖ Volume computation
- ❖ Measurement of area and volume
- ❖ Capacity
- ❖ Volume and area estimation
- ❖ Compound shapes
- ❖ Volume of combined solids
- ❖ Total length of edges of a solid
- ❖ Pythagoras's formula

Instructional strategies/Methods

- Investigations
- Project - based strategies
- Small group activity
- Problem based activity
- Guided instruction
- Discussion

Suggestions for Assessment

- ✓ Observation
- ✓ Flow charts
- ✓ Posters
- ✓ Graphic organizers
- ✓ Peer evaluation
- ✓ Performance tasks

Resources

- Manipulative
- Pattern blocks - Plane shapes
- Solid shapes
- Charts
- Computers

SUGGESTED TEACHING AND LEARNING ACTIVITIES

In this module, the following activities may be useful:

1. Estimate the Volume of Cylinders and Prisms

A useful way to start this module is to engage students in activity to develop their estimation skills. Estimation of volume presents opportunities for the student to investigate and experiment. Students could be asked to estimate how many small objects of known volume could fit into one large one whose volume is unknown. They can then confirm their estimates by doing actual experiments and measurements. This may be used to reinforce and teach different ways to compute volumes of solids.

Example: Students are put in groups and asked to estimate the volume of a given cylinder. One group might compare the cylinder with an appropriate cuboid, Diagram 1, and estimate the number of small cubes of known dimensions that can fit in the cuboid.

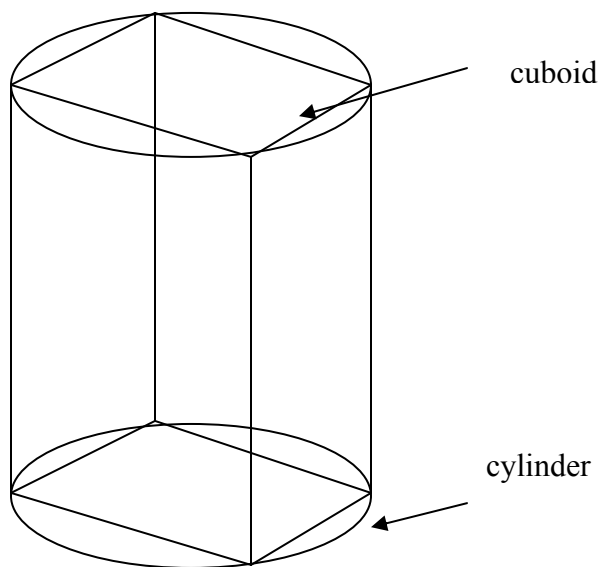


Diagram 1

Another group might choose to estimate the volume with 4 triangular prisms as shown in Diagram 2.

Groups can compare, discuss and evaluate their estimates. This is an excellent opportunity to achieve many of the outcomes in the Unit.

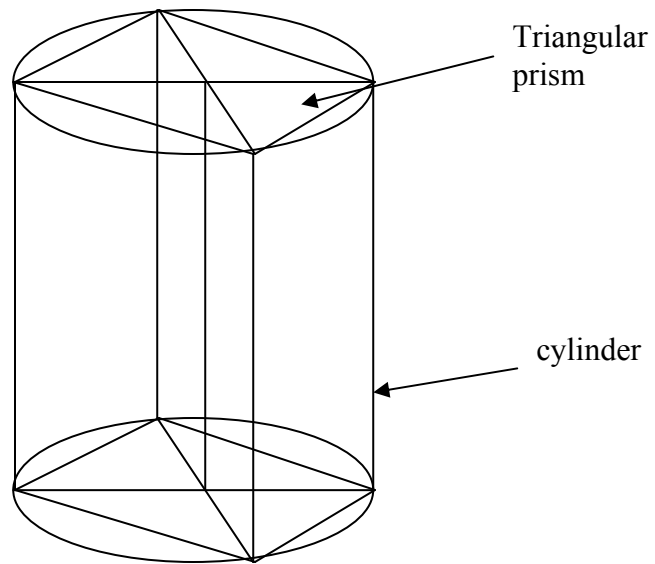
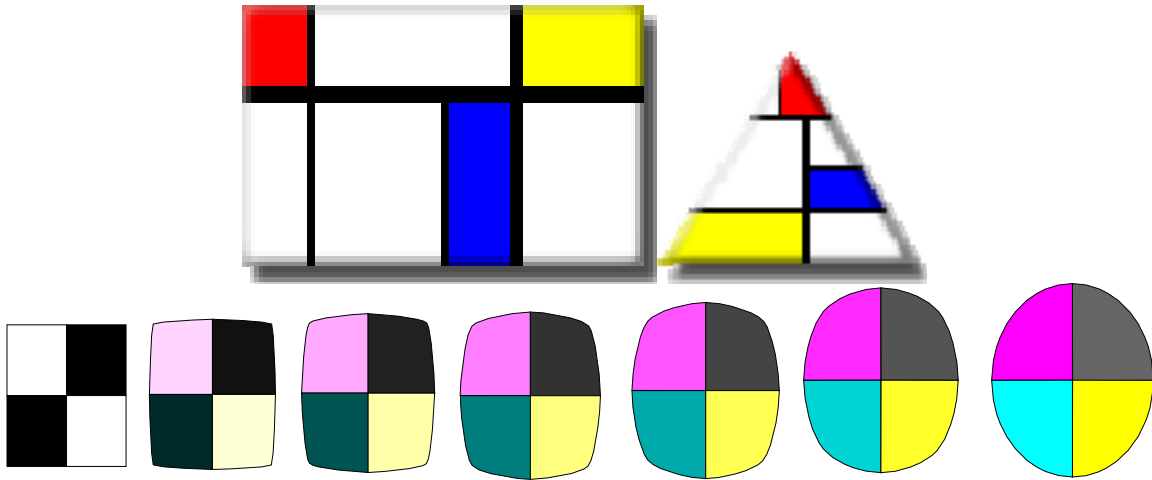


Diagram 2

Certain teachers may prefer to approach instruction through problem - solving. This could yield even greater rewards because of the opportunities it will create for developing critical thinking skills.

UNIT: 3



UNIT 3 - MODELLING SHAPES

GENERAL OUTCOMES

Students are expected to:

1. demonstrate understanding of circles, polygons and their properties;
2. show increased competence in the use of visualization, spatial reasoning and geometric modelling to analyze the characteristics of 2-dimensional and 3- dimensional objects;
3. demonstrate enhanced ability in problem- solving through the use of spatial skills;
4. use deductive reasoning to prove conjectures and communicate geometrical ideas clearly, formally and using a variety of tools;
5. appreciate the role of symmetry in understanding patterns;
6. use geometric models to represent simple transformations to gain insights into and solve problems in other areas of mathematics, other disciplines and other areas of interest such as art and architecture;
8. use precise mathematical language to communicate logical deductions and proofs of conjectures made about geometric figures;
9. appreciate the role of technology in gaining a better understanding of shape and space in general.

SPECIFIC OUTCOMES BY LEVELS

CORE

Students are expected to:

1. apply the properties of equality and parallelism of sides and equality of angles of triangles, quadrilaterals and regular polygons to solve problem;
 2. select and use appropriate geometrical and technological tools to:
 - construct line segments, angles and plane geometrical shapes (regular and irregular) for specific conditions.
 - bisect a given angle and line segment
 - construct line segments parallel to or perpendicular to a given line segment
 - draw plane shapes (regular and irregular) given specific conditions;
 3. use the properties of diameter, radius and chord, angles in the same segment, semi-circle, angles at the centre and circumference to prove conditions, perform computations and solve problems;
 4. draw and construct representations of 2-D and 3-D geometric objects using a variety of tools such as geometrical instruments and computer software;
 5. model transformations such as translation, reflection, and rotation on points in 3-dimensional objects;
 6. draw scaled diagrams to show the plan and elevation of cuboids, cylinders and cones;
 7. represent the plan and elevation of a cuboid, cube, cylinder, cone and sphere by drawing.
-

EXTENSIONS

Students are expected to:

9. use knowledge of similarity to do simple calculations of areas of triangle;.
 10. draw and construct representations of 2-dimensional and 3-dimensional geometric objects using computer software such as AutoCAD, Coral draw or any other suitable CAD software;
 11. represent the plan and elevation of a cube, cuboid, cylinder, cone and sphere on the computer;
 12. model transformations on given 3- dimensional shapes using CAD software.
-

Content

- ❖ Geometric constructions
- ❖ Plans and elevations
- ❖ Properties of polygons
- ❖ Angle properties of circles
- ❖ Diameter, radius and chord
- ❖ Transformations
- ❖ Symmetry
- ❖ Properties of shapes that enable tessellation

Instructional Strategies/Methods

- Cooperative learning groups
- Individual work
- Simulations
- Class discussions

Suggestions for Assessment

- ✓ Paper/pencil tests
- ✓ Oral presentations
- ✓ Construction models
- ✓ Performance appraisals from resource personnel
- ✓ Portfolios showing various outputs
- ✓ Presentations of designs using a variety of media.
- ✓ Project work

**Resources**

- Building plans
- Geometrical instruments
- Site visits
- Resource personnel from job related areas
- Appropriate computer software
- Templates of various 2-D and 3-D shapes
- Pictures

SAMPLE LESSON

Unit 1:

The Lizard and the Fly

Measure the shortcut the lizard makes to catch the fly

A farmer, at work in his tool shed, observes a lizard aiming to catch a fly. The lizard was at the middle of the front wall of the tool shed 30 centimetres away from the floor. The fly was resting at the centre of the back wall 30 centimetres from the horizontal ceiling.

Your task is to use your Mathematics classroom to model the farmer's tool shed and to use your model to determine the shortest distance the lizard must walk to reach the fly.

Maintaining Accurate Measurements

General Topic: Modelling Three-Dimensional Shapes

Specific Objectives: Students will be able to:

- Justify the use of a particular unit of measure for quoting a given quantity
- Measure quantities such as length to a given degree of accuracy
- Construct a scaled model of the net of the Mathematics classroom using analytical or technological methods
- Draw multiple paths to show how the lizard can walk to the mosquito
- Justify the choice of the lizard's path using geometrical or other ideas
- Determine distances on models in accordance with a given scale
- Develop co-operation and persistence as part of good team work.

Materials:

- Construction paper or computer with graphic software.
- Measuring tape
- Ladder
- Pair of compasses

Levels of Learning	Learning Mode	Essential Learning Outcomes
Knowledge : Conversion/ Metric and Imperial units	Constructivist Approach: Investigation	Communication Problem -solving
Comprehension: represent tool shed by Mathematics classroom	Multiple Intelligences: Visual/Spatial, Logical/Mathematical Interpersonal Skills	Technological Competence
Application: use of scales, geometrical and other ideas		
Evaluation: justify choice of shortest distance		

Assessment

Design a scoring rubric to highlight and reward the various aspects of the task.

Proposed Scoring Rubric Criteria

- accuracy in use of scales and scale conversions
- task completion
- accuracy of model created
- justification of choice of shortest distance with evidence
- presentation of results

STANDARDS

MODULE 5

Mathematics Standards

MODULE 5: TAKING CONTROL OF SPACE

At the end of Form 5, students will be able to:

1. Describe and apply relationships between numbers, their uses and their representations.

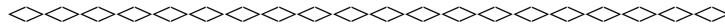
- 1.1 Make quick and rough estimates of quantities used in everyday life using appropriate equivalents between the Metric and Imperial systems of Units, justifying the use of a particular unit and giving answers to a given degree of accuracy;
- 1.2 Apply multiple representations of numbers such as integers, fractions, decimals, exponents and scientific notation in the creation and use of scales for map reading and accurate drawing of 2D and 3D geometrical figures, and appreciate that performing arithmetical operations on measurements affects precision and determines the units of the quantities used.

2. Estimate and compute mathematical quantities using multiple strategies.

- 2.1 Select and use the most efficient estimation and computational methods such as mental arithmetic, estimation strategies, paper and pencil techniques, and technology-supported methods:
 - (i) in the use of scales for map reading and accurate drawing of 2-dimensional and 3-dimensional geometrical objects;
 - (ii) in computing perimeter and area of 2-dimensional geometrical shapes and figures using measurement formulae;
 - (iii) in computing the areas of compound 2-dimensional shapes and the surface areas and volumes of 3-dimensional geometrical figures such as cubes, cuboids, cylinders, cones, spheres and right prism;
- 2.2 Appreciate that performing arithmetical operations on measurements, affects precision and should be stated with a given margin of error.

3. Estimate and measure to a required degree of accuracy and precision by selecting and using appropriate units and tools including technological tools.

- 3.1 Use appropriate measuring instruments, technological tools, and units such as length, time, temperature, mass, and degrees, in both the imperial and metric systems, to make estimates and accurate measurements in simple situations, to a required degree of accuracy and precision;
- 3.2 Make conjectures about the properties of 2-dimensional shapes such as circles and polygons and communicate these geometric ideas clearly using a variety of presentations;
- 3.3 Use appropriate measuring instruments and geometrical instruments to construct specific lines, angles and 2-dimensional shapes and use spatial reasoning to analyze the characteristics of both 2-dimensional shapes and 3-dimensional objects.



MATHEMATICS

3



THE MILLENNIAL LEARNER



ICT INFUSION IN THE CURRICULUM

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THE MILLENNIAL LEARNER

Today's rapidly changing technological and global environment has impacted significantly on learners in the classroom. Learners today have transcended traditional methods of learning and have entered the realm of what is now known as Millennial Learning.

Understanding the Millennial Learner

The millennial learner is:

- Special
- Global
- Technology-oriented
- Multi-tasking
- Problem- solver
- Independent thinker
- Instant
- Collaborative learner
- Borderless and limitless
- Trial and error learner
- Visual
- Team-oriented
- Creative
- Multi-faceted and multi-intelligent.



The Millennial Learner and Essential 21st Century Skills

It has become imperative that educators today integrate the Essential Learning Outcomes with the Essential 21st Century Skills as an integral part of their teaching if the Millennial Learner is to be nurtured in the classroom.

Some Essential 21st Century Skills:

- Personal and Social Responsibility - mindful of responsibility to authority and others;
- Adaptability – demonstrating flexibility;

- Creativity – generation of new ideas;
- Critical thinking – sound reasoning;
- Digital literacy and Collaborative skills – use of computer- Internet, Blogs and Wikis;
- Self-directed Learning – pursuing one’s own learning needs.

THE MILLENNIAL LEARNER AND DIFFERENTIATED INSTRUCTIONS:

Learners are differentiated by nature and Differentiated Instructions provides respect and attention to all learners.

Differentiated Instructions is a teaching model based on the premise that instructional approaches should vary and be adapted in relation to the individual and diverse learners in the classroom. The model of Differentiated Instructions requires teachers to be flexible in their approach to teaching and adjusting the Curriculum and presentation of content to learners rather than expecting learners to modify themselves for the Curriculum.

Adaptation of Content for Differentiated Instructions

Types of Adaptations:

1. **Volume of Content** – increase or decrease the number of Learning Outcomes.
2. **Time** – vary time on task for learning, task completion and testing.
3. **Level of Support** – use peer support to increase the amount of individual support to learners.
4. **Input** – Vary teaching and learning strategies through the use of different visual aids, concrete examples, hands-on activities, cooperative groups, independent work for higher level ability learners.
5. **Level of difficulty** - adapt skill level, task set and approaches to task
6. **Output**- adapt ways learners can respond to instructions e.g., verbal responses, demonstrations.
7. **Participation** – adapt extent of learner participation through brain- based activities such as reflections and peer sharing at opportune moments during activity and assisting in demonstrations.
8. **Alternate Pathway** – adapt goals and expected outcomes for a given set of resources.
9. **Tailor Curriculum** – provide appropriate instructions and materials to meet learners’ individual needs.

THE MILLENNIAL LEARNER AND BRAIN-BASED ACTIVITIES IN THE CLASSROOM

The processing of new information by the human brain can be described using the stages shown in the table below:

Memory Storage	Duration of Processing Time
Sensory Register	milliseconds
Immediate Memory	30 seconds
Working Memory	20 minutes
Long Term Memory	24 hours

The implication of the Brain-Based model for classroom activities ideally requires reinforcement intervals at the end of twenty minute intervals in the lesson. Lesson Plans can be adapted to contain twenty- minute segments to facilitate information transfer on what is learnt, to long term storage and to give meaning to learning activity.

Short Reflection Intervals may include:

- Discussion or questioning, cognitive in nature, for Left Brain engagement
- Writing about thoughts and feelings on learning activity (affective domain) for Right Brain engagement
- Completion of questionnaire.

Active attention and engagement of learners may be enhanced by incorporating:

- Multiple perspectives of topic and varying learning styles
- Multi sensory opportunities
- Appealing sound and music
- Visuals
- Power of choice – identify areas of interest and strength of students
- Humour
- Activating Prior Knowledge to determine suitable starting point for lesson
- Anticipation
- Participation and collaboration
- Proximity to learners
- Self-direction
- Capturing teachable moments.

Parallel Processing is second nature to the millennial learner whose environment is one which is exponentially growing in digital information. Brain-based activities reduce the discongruence between classroom experiences and experiences in the real world and enhance both the process and product of learning.

ICT INFUSION IN THE CLASSROOM

The millennial learner is immersed in a rapidly changing technological environment. The development of technological competence has taken on greater significance in the classroom today.



Benefits of technological integration:

- Greater motivation
- Development of higher-order thinking skills
- Enhancement of communication and collaboration skills
- Transfer of skills to work environment and lifelong learning
- Enhanced problem-solving skills

Adaptation of curriculum for ICT infusion:

- Varied presentation of information to learners
- A varied selection of resources and tools to learners
- Facilitates higher order thinking by reduction of tedious algorithmic procedures
- Availability of a wider range of information sources to learners
- Posting of teaching/learning instructions on a wiki site
- Assessment to include digital presentations

Forms of ICT for the classroom:

- Calculators
- Software packages
- Digital monitoring devices e.g. sensors, global positioning systems
- Digital cameras
- Email
- Digital video equipment

Project- based learning is a learner-centred model for teaching and learning. It allows the learner to make choices in the selection of content areas including the exploration of authentic contexts. Project- based learning promotes “learning by doing” and gives deeper meaning to learning. Learners of differentiated abilities are catered for and their creativity and motivation skills enhanced.

FOCUS: Project-Based Learning

Definition:

Project-Based Learning is an individual or group activity that goes on over a period of time, resulting in a product, presentation, or performance. It typically has a timeline and milestones and other aspects of formative evaluation as the project proceeds.

Brainstorming

- Organizing the brainstormed ideas
- Developing a draft
- Obtaining feedback
- Revising, which may involve going back to earlier steps
- Publishing

Here are some general ideas about Project-Based Learning:

1. Project-based learning is learner-centred. Students have a significant voice in selecting the content areas and nature of the projects that they do. There is considerable focus on students understanding what they are doing, why it is important and how they will be assessed. Indeed, students may help to set some of the goals over which they will be

assessed and how they will be assessed over these goals. All of these learner-centred characteristics of PBL contribute to learner motivation and active engagement. A high level of intrinsic motivation and active engagement are essential to the success of a PBL lesson.

2. From the student's point of view, Project-Based Learning:

- Is learner centred and intrinsically motivating
- Encourages collaboration and cooperative learning
- Requires students to produce a product, presentation, or performance
- Allows students to make incremental and continual improvement in their product, presentation, or performance
- Is designed so that students are actively engaged in "doing" things rather than in "learning about" something
- Is challenging; focusing on higher-order skills.

3. From the teacher's point of view, Project-Based Learning:

- Has authentic content and purpose
- Uses authentic assessment
- Is teacher facilitated; but the teacher is much more a "guide on the side" rather than a "sage on the stage."
- Has explicit educational goals
- Is rooted in Constructivism (a social learning theory)
- Is designed so that the teacher will be a learner
- The teacher plays a major role in setting the learning goals of the project
- The teacher and students provide formative evaluation
- Teacher, students, and others may help in the summative (final) evaluation
- Rubrics are created by a combination of teacher and students. These facilitate self evaluation, peer evaluation, evaluation by the teacher and evaluation by outside experts.

4. From a research point of view, Project-Based Learning is supported by work in:
- Constructivism
 - Situated Learning Theory
 - Motivation Theory
 - Inquiry & Discovery-Based Learning
 - Cooperative Learning
 - Individual & Collaborative Problem Solving
 - Peer Instruction
 - Problem-Based Learning

ACTIVITY 1:

Working in small groups, share your experiences (both successes and failures) in making use of Project-Based Learning in Mathematics education. As you think about this activity, you might conclude that PBL is used much more in non-Math disciplines than in Math education. Why do you think this is the case?

ICT-Assisted PBL in Math Education

As noted elsewhere in this work, there are many possible goals for Mathematics education. These goals can be expressed as a quite specific scope and sequence, such as textbook series tend to do.

In addition to a scope and sequence approach, one can look at some guiding themes and principles. Three quite general areas of expertise that might be developed by a person studying Mathematics are as follows:

1. Mathematics as a human endeavour. Mathematics has a very long history. Mathematics has beauty. Mathematics is an important aspect of aspect of past and current cultures. Mathematics is "the queen of the sciences."
2. Mathematics as an interdisciplinary language and tool. Mathematics can be used to help represent, communicate about, and solve problems in many different disciplines. Many jobs and other aspects of responsible adult life in our society require some mathematical knowledge and skills.
3. Mathematics as a discipline. The formal study of and research in Mathematics is at least 5,000 years old. It is a deep and wide discipline with a huge amount of accumulated knowledge.

Each of these three general areas of mathematics expertise lends itself to both ICT-Assisted Project-Based Learning and ICT-Assisted Problem-Based Learning. The next three subsections give a few examples.

TECHNOLOGY INTEGRATION STRATEGIES

Integrating technology into the classroom has become an imperative for teachers at all grade levels. State standards require it and research supports its positive impact on student learning. Nearly all schools today have computer labs or a computer in the classroom and many also have Internet connections.

Teachers know that they must integrate technology into their lessons and they finally have the equipment at their disposal. Understanding why it should happen and how best to do it are often less clear.

Reasons for Integrating Technology

When done effectively, technology has a positive impact on student learning. It can:

- Increase student motivation for learning
- Improve communication of learning goals
- Facilitate higher-order thinking skills
- Build valuable skills that students will use in college and in the workplace
- Expand students' understanding from novice to mastery

There is no denying that computer technology has become as commonplace as the telephone in American society. To make learning relevant to students, this reality needs to be acknowledged in the form of technology-based lessons.

How Technology Changes the Way Students Learn

Students still learn the same way that they always have: by comparing new information with previously acquired knowledge and skills. However, students today have different preferences for how information is presented, based on their experiences with computer technology.

What is different in the learning process is the array of tools available to the learner. Computer technology automates previously laborious processes, allowing students to focus on developing higher- order thinking skills. It also makes an unprecedented number of information resources available to students, often making for a more circuitous path to learning. Students use hypermedia (linked information) to access non-sequential pieces of information. Read "Click Here: Teaching the Next Generation" to learn more about how to appeal to these learning styles.

ACTIVITIES FOR ALL CLASSROOMS

There are literally thousands of ideas for using technology in your classroom. Online tools can be used for collaboration; the Internet provides a rich source of information; and multimedia tools allow students to assemble presentations that are both educational and entertaining.

The following ideas can be used in virtually any classroom, regardless of course content.

- **Create a Class Website**—The ‘Web’ is an excellent way to communicate with your students and their parents or guardians. You can include course information, assignments, lecture notes and presentations, links to interesting sites, challenges, study tools, links to textbook websites and many other features.
- **Take Your Class on a Virtual Field Trip**—Use the Internet to visit one of the many online exhibitions available. Notable institutions, such as the Library of Congress and the Smithsonian Institution, have exhibitions and guides designed especially for teachers and students. When funds or time do not allow a field trip, look online to find a worthwhile destination.
- **Take Part in a ‘Web’ Event**—These online events allow classes to observe and interact in educational activities occurring in real time. Using chatroom technology or streaming video, these events are exciting to both students and educators. Students can often ask experts questions and can read (or hear) what other students from around the world are asking.
- **Create a WebQuest**—This popular idea began back when the Internet was in its infancy and it has grown by epic proportions ever since. Designed to engage students in Internet-based tasks that require higher-order thinking skills, WebQuests can range from a simple one class period activity to an extended group project that can take all semester. You can create your own using a template or visit one of the many sites that have teacher-created WebQuests ready for your use.
- **Visit Your Textbook Website**—Many publishers offer supplementary Websites to accompany textbook study. Glencoe/McGraw-Hill offers Websites with extensive resources as a supplement to the textbook program. Glencoe Online Learning Centres have a wide array of features including tools for self-study, chapter summaries and links, video clips, and interactive activities.
- **Participate in an Online Research Project**—The Internet is filled with ongoing projects that allow students to contribute by collecting, submitting and analyzing data, and submitting ideas, or contributing work online.
- **Have Students Create a Multimedia Presentation**—Ask students to use various digital media, such as digital video clips, audio clips, and digital photographs to assemble a multimedia

presentation. Use your content standards to identify choices that students can make about the project topic. The end product could result in a Website, PowerPoint® presentation, or other hypermedia product.

- **Use Common Productivity Software for Teaching and Learning**—Software that is commonly used to increase productivity in offices and in homes can also be effectively adapted to school use. Common products, such as word processing software, spreadsheets, and presentation-making applications, have a multitude of uses in the classroom. Whether used to organize data collected in a science project, to track revisions in the writing process, or to create a timeline of events, these software products can increase student motivation by making tedious tasks easier.

ADDITIONAL FORMS OF TECHNOLOGY FOR THE CLASSROOM

There are numerous other forms of hardware and software that can help engage students in technology-assisted learning. These include:

Peripheral or Course-Specific Technology

- Calculator-Based Labs™
- Calculators
- Scientific monitoring devices, such as probes and sensors
- Drawing software
- Animation software
- Global Positioning Systems
- Email
- Digital cameras
- Digital video editing suites

TECHNOLOGY LESSON GUIDELINES

Whichever tools you choose to use in your classroom, you need to remain focused on the purpose of each activity. Use the following tips to guide you in your technology integration efforts and you will be on your way to success.

- Clearly define learning objectives for each lesson.
- Align lessons with content standards.
- Prepare learning assessment tools in advance.

- Share assessment methods with students.
- Familiarize yourself with the technology before using it in the classroom.
- Be prepared with a back-up lesson in case technology malfunctions.

The foregoing article was adapted from the work of Elizabeth Melville, an eLearning consultant and editor of Teaching Today.

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SAMPLE – PROJECT BASED LEARNING

TOPIC: GEOMETRICAL TRANSFORMATIONS

The Project ‘Motion Geometry’ gets students involved in both research practical activities to explore the motion of two-dimensional and three-dimensional objects in Geometry. The Unit employs the Jigsaw Model of Cooperative Learning. All students will research the meaning of the term Transformation, identify the four types of transformations and give a real-life example of each. Students, working in flexible groups, will conduct hands-on explorations on Translation, Reflection, Rotation and Enlargement using resources provided.

Each group’s presentation will be a part of the Jigsaw Model to complete the Unit. The Essential Question ‘Why do we need to study change?’ provides a breadth of geometrical experiences in two and three dimensions and allows for deeper understanding of the topic through the development of spatial awareness, geometric intuition and the ability to visualize. The Unit inspires 21st Century Skills such as Self-Direction, Creativity and Intellectual Curiosity, Interpersonal and Collaborative Skills and Media Literacy Skills. This Unit is open to different approaches but the practical approach put things in a way that is stimulating and engaging in a child- centred learning environment.

Level: Form 4

Approximate Time Needed: Two weeks (10 40-minutes periods)

Content Standard:

Students will apply the properties of Translation, Reflection, Rotation and enlargement to two-dimensional and three-dimensional geometrical objects.

Learning Outcomes:

1. Define terms: Transformation, Reflection, Translation, Rotation and Enlargement;
2. Investigate real- life applications of Reflections, Translations, Rotations and Enlargements and record suitable examples using illustrations;
3. Design activities to explore the properties of Reflections, Translations, Rotations and Enlargements in Jigsaw Model Cooperative learning;
4. Extend activity to explore transformation of a creative combination of two or more transformations;

5. Represent the objects and their transformations diagrammatically;
6. Use multimedia software to create slides to visually support oral presentation of project.

Project questions

- Why do we need to study change? – Big Idea
- How can we make changes using shapes?
- What is meant by the term Transformation?
- What are the different types of transformations?
- How can I perform and represent each type of transformation?
- What are real-life applications of Reflections, Translations, Rotations and Enlargements?

Assessment Timeline

Before project work begins		Students work on projects and complete tasks		After project work is completed	
Brainstorm ideas	Meaning of the term: Transformation			Multimedia Oral presentation of both Interim and Activity Reports	Project Report Scoring Rubric
Research Using Internet • Interview	Definition Transformation Types and their definitions, Real-life examples Interim Project Report - rubric Data to gauge support needed Digital skills – Internet access, Multimedia, Wiki, Blog reflection	<ul style="list-style-type: none"> Teacher Observation and Teacher-student Conferences Monitoring Checklist for design and conduct of Activity, Teamwork, media skills – reference to Teacher postings of Instructions on wiki, Blog Collaboration Reflective Journal 	<ul style="list-style-type: none"> Formative-Immediate feedback Appropriate resources selected/focus on task/time on task/willingness to share ideas and availability of activity station 	Reflective Journal entries	Rubric

Assessment	Process and Purpose of Assessment
Brainstorm Ideas	Teacher obtains information to determine starting point of project
Interim Project Report	Students conduct Internet research on topic <i>Transformations</i> to include the four types of transformations, definitions and real-life applications of each- presented as part of Final Project Report.
Interview	Teacher obtains information on readiness of students to embark upon project and their digital literacy skills in order to plan support needed for both Activity and Project Report.
Direct Observation and Teacher – Student Conferences	Teachers use questioning strategies to monitor student progress, probe for understanding, and engage students in higher-order thinking. Teachers question students when circulating during group and individual work time, as well as during conferences. Teacher also redirects student to Curriculum-Framing Questions throughout the project to analyze student understanding.
Monitoring checklist	Criteria assessment, anecdotal notes from observations and interactions with individuals and groups and from the conferences, form part of practical activity assessment.
Collaboration with peers and Teacher	Blog entries, Teacher posted wiki
Oral Presentation Content Scoring Guide	Graded Criteria
Oral Presentation Scoring Guide	Graded Criteria
Reflection Journal	Students reflect upon what they have learned in the Unit, returning to the Essential and Unit Question, <i>“What happens when things move?”</i> Students cite evidence from their research and their practical activities. The teacher uses these reflections to assess students’ growth throughout the Unit.

ASSESSMENT DETAILS

Assessment Summary

After initial brainstorming of essential question and concept of transformation, students will research definitions and real-life applications of the types of transformation and make a brochure of findings which will be assessed using a scoring rubric. Individual interviews will provide formative feedback on level of digital support needed for conducting activities, using collaborative wiki and blog sites, and reporting project. The teacher’s Direct Observation will serve to guide activity, render support needed in a timely manner and gauge students’ thinking. The checklist provides quick assessment on monitoring students’ progress during the project, allowing teacher to maximize facilitation of groups. Students’ blog entries will be used to assess their reflection during the project. Collaboration criteria ensure that students engage in Internet,

use which is an integral part of the project design. The multimedia presentation of the project report and the extension component enhances communication and digital skills and allows for creativity.

Prerequisite Skills

Knowledge of two-dimensional and three-dimensional geometrical shapes, Working knowledge of Cartesian plane and axes, basic Computer Literacy – insert/draw shapes, use of internet, Collaborative blog and wiki, PowerPoint software.

Instructional Procedure

What the Teacher Will Do	Eliciting Questions/Student Responses	What the Students Will Do
<p>Brainstorm Session</p> <p>The teacher will initiate discussion on</p> <ol style="list-style-type: none"> 1) Changes we experience in our lives– weather, body changes, mood swings, global meltdown, new class, 2) The teacher will introduce the Essential Question ‘Why do we need to study change ?’ by considering briefly the impact of changes on an object itself? <p>Students have the ability to effect changes themselves – be change agents in individual lives, classroom, school, home, community, nation and global environments.</p> <p>Teacher probes class for synonym/meaning of ‘CHANGE’ leading to terminology ‘TRANSFORMATION’ and engages students to come up with types of transformations – Translation, Reflection, Rotation and Enlargement.</p> <p>Teacher assigns research work on types of transformation and their applications in real-life situations to be assessed as a part of the Project Report – Oral Presentation Scoring Rubric.</p> <p>Teacher supplies Oral Presentation Scoring Rubric to students.</p> <p>Teacher organizes Workstations for the Activities of the Project and posts up the initial instructions on the teachers wiki for the class.</p>	<p>Suggestions</p> <p>Share ideas with peers</p> <p>Contemplates how they can effect changes throughout their lives, what they can do to empower themselves-</p> <p>Implication for lifelong learning</p>	<p>Reflections entries</p> <p>Conduct Research using Internet</p> <p>Organise presentation using PowerPoint seeking assistance for media support when need arises</p> <p>Students refer to wiki site and select activity of first choice.</p>

<p>WORKSTATION - TRANSLATION</p> <p>Resources</p> <p>2-D and 3-D geometrical objects</p> <p>Translucent paper</p> <p>Grid paper</p> <p>Geo board and rubber bands</p> <p>Bristol Board</p> <p>Sheets of coloured paper</p> <p>Pair of scissors</p> <p>Pencil, paper, ruler</p> <p>Computer and printing facility</p> <p>ACTIVITY OUTLINE SHEET – TRANSLATION</p> <p>WORK STATION - REFLECTION</p> <p>Resources</p> <p>2-D and 3-D geometrical objects</p> <p>Translucent paper</p> <p>Grid paper</p> <p>Geo board and rubber bands</p> <p>Bristol Board</p> <p>Sheets of coloured paper</p> <p>Pair of scissors</p> <p>Pencil, paper, ruler</p> <p>Learning Resource- Reflector and transparent 2-D and 3-D geometrical objects</p> <p>Computer and printing facility</p> <p>ACTIVITY OUTLINE SHEET –REFLECTION</p>		<p>Students collaborate on blog to match flexible groups with activity and select a group of choice.</p> <p>Students work in groups on activity. They select appropriate resources and design activity as outlined in ACTIVITY SHEETS placed at the station of their choice.</p> <p>Students share knowledge and skills in orderly way upon request made to them by other students.</p> <p>At the end of activity, students engage in blogging activity to share ideas and make decisions about adjusting activities.</p> <p>Students make blog entries on the progress of their activities to facilitate availability of work-stations for switching of activities</p>
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<p>WORKSTATION - ROTATION</p> <p>Resources 2-D and 3-D Geometrical Objects Translucent paper Grid paper Geo board and rubber bands Bristol Board Sheets of coloured paper Pair of scissors Pencil, paper, ruler, protractor, string, thumb tack Computer and printing facility</p> <p>ACTIVITY OUTLINE SHEET –ROTATION</p> <p>WORK STATION - ENLARGEMENT</p> <p>Resources 2-D and 3-D geometrical objects Translucent paper Grid paper Geo board and rubber bands Bristol Board Sheets of coloured paper Pair of scissors Pencil, paper, ruler Computer and printing facility</p> <p>ACTIVITY OUTLINE SHEET –ROTATION</p> <p>Teacher posts Name of Workstations, Resources and an outline for the Activity on wiki site for class. Students view site and sign up for the activity of their first choice.</p> <p>Teacher ensures all activities are distributed and facilitates formation of flexible groups.</p> <p>Teacher facilitation and checklist monitoring continues for duration of project activities.</p>		<p>Students checks wiki site for adjustments to activities made by the teacher.</p> <p>Students make interim recordings for each activity which would contribute to project presentation.</p> <p>Students share ‘expertise’ on completed activities upon request.</p> <p>Students prepare Project Report. Students collaborate with groups and make decision on which aspect of Activity Report would be most suitable to select for oral presentation as part of Jigsaw Puzzle Cooperative Group Work.</p>
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ACCOMMODATIONS FOR DIFFERENTIATED INSTRUCTIONS

Students with different levels of learning difficulties

Cooperative Learning Flexible Grouping, Alternative assessments, Varied time on task

Non-Native English Speaker

Dual-language dictionary, Interviews to determine prior knowledge and attainment of Learning Outcomes, Visuals, Interpretative listening and monitoring of understanding, Paraphrasing and opportunity for vocalising.

Gifted Students

Independent learning activities, Challenging tasks, Higher Order thinking skills, Meta cognitive skills.

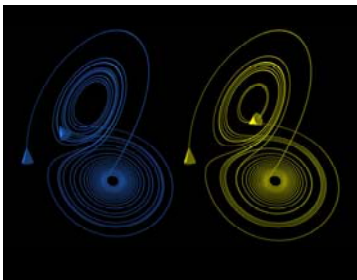
STUDENT SAMPLE

UNIT: MOTION GEOMETRY -
Transformations
By Indra Dass

TRANSLATION

- A Translation “slides” an object a fixed distance in a given direction.
- The original object and its translation have the same shape and size.
- The original object and its translation face in the same direction.

Real-life Application of Translation



REFLECTION

- A Reflection can be seen in water or in a mirror.
- An object and its reflection have the same shape and size
- An object and its reflection face in opposite directions

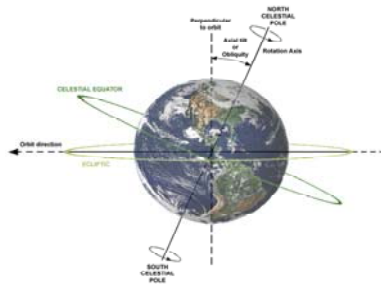
Real-life Application of Reflection



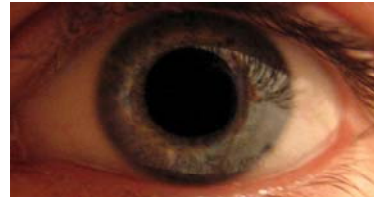
ROTATION


- A Rotation is a transformation that turns a figure about a fixed point called a Centre of Rotation
- An object and its rotation are the same shape and size
- . An object and its rotation may be turned in different directions.

Real-life Application of Rotations



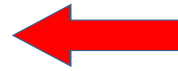
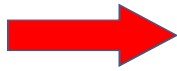
Real-life Application of Enlargements



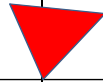
	
<p>TRANSLATION</p>	

ENLARGEMENTS

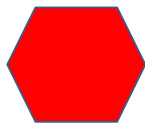
- An Enlargement changes the size of a shape by multiplying all its proportions by the same scale factor.
- An Enlargement may be larger or smaller than the original object.
- A scale factor greater than 1 will give a new shape that is larger than the original shape.
- A scale factor less than 1 will give a new shape that is smaller than the original shape.



REFLECTION



ROTATION



ENLARGEMENT

PROJECT REPORT- ASSESSMENT
Scoring Rubric PowerPoint Presentation

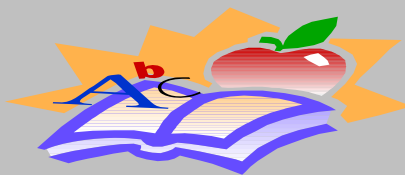
Specific Expectations Content:					
Correct definition; Real-life examples with picture; Representation of result of Activity/ Details					
Translation	1	2	3	4	5
Reflection	1	2	3	4	5
Rotation	1	2	3	4	5
Enlargement	1	2	3	4	5
General Expectations:					
Slide layouts are neat, well organised	1	2	3	4	5
Correct spelling, grammar, punctuation	1	2	3	4	5
Overall appearance, interest level	1	2	3	4	5
Extension Activity – Design and Creativity					
Displays great skills of contrast, balance, direction, centre of interest, authentic					4
Applies contrast, balance, direction, centre of interest, use of source material as a starting point					3
Tries to apply contrast, balance, direction, centre of interest, copied design					2
Does not appear to be able to apply most design principles to work, not make much effort to meet requirement					1
Collaboration with Peers					
Almost always listens to, shares work and supports the efforts of others in the group. Tries to keep students working.					4
Usually listens to, shares work and supports the efforts of others in the group. Does not cause 'waves' in the group.					3
Often listens to, shares work and supports the efforts of others in the group. Sometimes not a good team member.					2
Rarely listens to, shares work and supports the efforts of others in the group. Often is not a good team member.					1
Oral Presentation					
Showed full understanding of project, spoke clearly and distinctly all the time.					4
Showed good understanding of project, spoke clearly most of the times.					3
Showed good understanding of some parts of the project, mispronounced some words					2
Does not seem to understand the topic, speech unclear at times.					1

MATHEMATICS

4



GLOSSARY OF MATHEMATICAL TERMS



A

Absolute Value

Absolute value makes a negative number positive. Positive numbers and 0 are left unchanged. The absolute value of x is written $|x|$. We write $|-6| = 6$ and $|8| = 8$.

Accuracy

How close an approximation is to an actual value

Acute Angle

An angle the size of which is between 0 and 90 degrees

Additive Inverse of a Number

The opposite of a number. For example, the additive inverse of 12 is -12 . The additive inverse of -3 is 3. Formally, the additive inverse of x is $-x$. Note: The sum of a number and its additive inverse is 0

Algebraic Equations

Mathematical statements involving an equal sign and at least one variable

Algebraic Expressions

A set of numbers, symbols, and/or letters connected by '+' or '-' signs, e.g., $3x + 2y - 5$

Algebraic Term

A part of a polynomial, e.g., in the expression $5x^2 + 3x + 1$, $5x^2$, $3x$, and 1 are all terms

Algorithm

A systematic step-by-step process that can be followed in arriving at conclusions

Amount

The sum of money received or repaid on an investment or loan at the end of a specific period

Angle

A measure of how much something has turned

Anticlockwise

Opposite the direction in which the hands of a clock move

Appreciation

The increase in value of some asset

Approximation

Any amount that is close to, but is not the exact amount

Arc

A line that connects two points along the *circumference* of a circle

Area

A measure of the amount of surface of a plane shape

Arithmetic Mean

The most commonly used type of average. To find the arithmetic mean of a set of n numbers, add the numbers in the set and divide the sum by n .

Array

An ordered set of data

Axes

Reference lines on a graph

Axis of Reflection

The "mirror line" of a reflection, that is, the line across which a reflection takes place

Axis of Symmetry

A line of symmetry for a graph. The two sides of a graph on either side of the axis of symmetry look like mirror images of each other.

B**Bar Chart****Bar Graph**

A graph that uses separate bars of different lengths to show and compare data

Base in an Exponential Expression

a in the expression a^x . For example, 2 is the base in 2^3 . Similar to the base of a logarithm.

Bearing

Two similar ways of indicating direction. On the left below is a kind of bearing which uses compass points. The bearing S34°E means the direction is 34° away from due south directed towards the east. The other way, on the right below, measures the angle clockwise from due north.

Binary

Base two

Binomial

A polynomial with two terms which are not like terms. The following are all binomials: $2x - 3$, $3x^5 + 8x^4$, and $2ab - 6a^2b^5$

Bisect

Cut into two equal parts

C

Capacity

The internal volume of a container, i.e., the amount a container will hold when filled

Cardinal Numbers

The numbers 1, 2, 3, . . . as well as some types of infinity. Cardinal numbers are used to describe the number of elements in either finite or infinite sets

Cartesian Coordinate

The position of a point on a grid can be given by a pair of numbers, which are its distances from two lines called the *x-axis* and the *y-axis*. The first point on the *x-axis* is called the *x coordinate* and the second on the *y-axis* is called the *y-coordinate*

Cartesian Plane

The plane formed by a horizontal axis and a vertical axis, often labelled the **x-axis** and **y-axis**, respectively

Centre of Enlargement

The point from which lines are drawn to construct the enlargement of a diagram

Centre of Rotation

The fixed point from which rotated points move so that they stay the same distance away

Circle

The path traced out by the set of all points in a plane at a fixed distance from a fixed point called the centre

Circumference

The perimeter or distance right around the edge of a circle

Clockwise

The direction in which the hands of the clock move

Closure

A set is closed under an operation when any two elements of the set combine, using the operation, to give another element belonging to the set

Commission

Percentage of a business transaction paid to the agent for his/her work

Commutative Operation

Any operation \oplus for which $\mathbf{a} \oplus \mathbf{b} = \mathbf{b} \oplus \mathbf{a}$ for all values of \mathbf{a} and \mathbf{b} . Addition and multiplication are both commutative. Subtraction, division, and composition of functions are not. For example, $5 + 6 = 6 + 5$ but $5 - 6 \neq 6 - 5$

Complementary Angle

Two angles that add up to 90 degrees

Composite Number

Whole number that is composed by multiplying two other whole numbers. The dots of composite numbers can be lined up to form rectangles.

Compound Shape

A shape made up of more than one simple geometrical shape

Congruent Triangles

Triangles that are identical in every dimension. They have same shape and size.

Constant

A number or variable that maintains a fixed value

Continuous

When a variable can take any number in an interval it is said to be continuous.

Convert

Change to another form

Coordinate Plane

Refer to Cartesian plane

Cube

A solid that has *squares* for all of its faces

Cuboid

A solid that has *rectangles* for all of its faces

D**Data**

Name given to a collection of numerical facts or information

Decimal

A number written in base 10 notation

Denary

Base 10

Depreciation

The reduction in value of some asset

Determinant

A single number obtained from a matrix that reveals a variety of the matrix's properties. Determinants of small matrices are written and evaluated as shown below. Determinants may also be found using expansion by co-factors.

Diameter

Any straight line that joins two points on the circumference of the circle and passes through the centre of the circle

Digit

Any of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 used to write numbers. For example, the digits in the number 361 are 3, 6, and 1

Dimension

The dimension of a space is the number of coordinates needed to fix the position of a point in it

Direct Proportion**Direct Variation****Directly Proportional**

A relationship between two variables in which one is a constant multiple of the other. In particular, when one variable changes, the other changes in proportion to the first.

Discount

The amount of money deducted from the marked price of an item, so that a lesser price is paid for it

Discrete

A variable is said to be discrete if it can take just whole number values.

Disjoint Sets

Sets that have no members in common. The intersection of the sets is the empty set.

Distribution

The multiplication of one term by all the terms of an expression. The reverse of factorization.

Domain

The domain of a function is the set of elements on which the function operates.

E

Edge

The line where two faces of a solid meet

Element of a Matrix

One of the entries in a matrix. The address of an element is given by listing the row number, then the column number.

Elimination

A method used to remove one variable from two equations in order to find a value for one of them

Empty Set

A set that has no elements in it. It is symbolized by \emptyset or $\{ \}$.

Enlargement

A transformation that changes the size of an object without changing its shape

Equal Sets

Sets that contain exactly the same elements, no matter the order

Equidistant

Equally distant. For example, any two points on a circle are equidistant from the centre

Equivalent Sets

Sets with the same number of elements, no matter the elements

Estimation

An answer that is close to the exact answer and may be found by “rounding off” or applying approximations

Evaluate

To figure out or compute. For example, "evaluate $12 + \sqrt{25}$ " means to figure out that the expression simplifies to 17.

Expand

To multiply out the parts(factors) of an expression. See *distribution*.

Exponent

x in the expression a^x . For example, 3 is the exponent in 2^3 .

Exterior Angle

Angle formed when the side of a polygon is extended beyond the vertex. It is the angle that is formed between the extended side and the adjacent side.

F**Face**

A distinct surface of a solid shape

Factor

A whole number that divides into other whole numbers exactly

Factorization

The method used to express a polynomial expression as a product of its factors

Finite Set

A set that contains a countable number of elements

Formula

A general rule used to show the connection between related quantities

Fraction

A part of a whole. It is commonly represented by a number less than 1

Frequency

The number of times a particular item or event occurs when collecting data

Frequency Polygon

A line graph that is formed when the mid-points of the tops of the bars in a *histogram* are joined

Frequency Table

A tabulation of data with the matching frequency with which each value occurs

Function

A rule connecting two sets such that for each item in the first set, there is just one item that it is related to in the second set

G**Gradient**

The steepness or slope of a line

Graph

A diagrammatic representation of data or information

H**Highest Common Factor (HCF)**

The largest number that can divide all of a set of numbers exactly

Hire Purchase

A system of paying part of the cost of an item and then paying the rest in equal monthly or weekly amounts called instalments.

Histogram

A frequency graph similar to the bar graph but with the bars touching along their sides, and in which the area of each bar, rather than the height of each bar, gives the frequency.

Hypotenuse

The longest side of a right-angled triangle. It is the side that is opposite to the right angle.

I**Identity Element**

A number, that when combined with another number under an operation, remains unchanged.

Image

The result when an object is transformed

Image Point

A point after it has undergone a transformation

Imperial System

Traditional units of measurement, e.g., inches, feet, pounds, tons

Index

A number or variable placed as a superscript to the right of another number or quantity, indicating the number of times that the number or quantity is to be multiplied by itself (e.g., for 2^4 , 4 is the index). [See also *power*]

Inequality

A statement showing the comparison between two quantities. Such a comparison usually involves the use of the inequality signs: $<$, \leq , $>$, \geq , e.g., $2x \geq 8$.

Infinite Set

An infinite set is one that contains a countless number of elements

Interior Angles

The angles formed at the vertices inside a plane figure

Intersection

The intersection of two sets A and B is the set that contains elements that are common to both sets. The intersection of sets is shown by using the symbol \cap (e.g., $A \cap B$).

Inverse Element

Under a particular operation, it is the number that combines with a selected number to give the *identity element* for the operation.

Investment

The use of money to make a profit.

Irrational Number

A number that cannot be written in the rational form $\frac{a}{b}$, where a and b are integers and b is not zero, e.g., $\sqrt{7}$.

K**Kite**

A quadrilateral with two pairs of equal sides and one *line of symmetry*.

L**Line Graph**

A graph formed by plotting points and connecting these points to form a line

Line of Symmetry

A line about which a shape can be reflected onto itself

Line Segment

Part of a straight line between two given point.

Linear Equation

An equation in which the highest degree or power of any variable is 1

Linear Programming

An algorithm for solving problems asking the largest or smallest possible value of a linear polynomial. Any restrictions on the problem must be expressed as a system of inequalities; in particular, all equations and/or inequalities must be linear.

Loss

The difference in value when an item is sold for less than what was paid for it

Lowest Common Multiple (LCM)

The smallest number that any number from a specified set of more than one number can divide exactly

Low Quartile**Lower Quartile****First Quartile, Q_1**

For a set of data, a number for which 25% of the data is less than that number. The first quartile is the same as the median of the data which are less than the overall median. Same as the 25th percentile.

M**Magnitude**

The amount of a quantity Magnitude is never negative.

Manipulatives

Tactile resources that are used to facilitate teaching and learning

Mapping

A *relation* in which every element from the first set or domain relates to an element in the range

Mass

A measure of the amount of matter obtained in an object

Mean

This is obtained by dividing the sum of all the data by the number of items of data (also called *arithmetic mean*)

Median

The middle value (or mean of the two middle values) of a set of data once the data have been arranged in order of size

Mode

The most frequently occurring item

Model (Mathematical Model)

An equation or a system of equations representing real-world phenomena. Models also represent patterns found in graphs and/or data. Usually models are not exact matches of the objects or behaviour they represent. A good model should capture the essential character of whatever is being modelled.

Multiple

A multiple of a number n is any number that n divides exactly, i.e., $k \times n$ where k is a counting number.

Multiplicative Inverse of a Number (Reciprocal)

The reciprocal of x is $\frac{1}{x}$. In other words, a reciprocal is a fraction flipped upside down. Multiplicative inverse means the same thing as reciprocal.

N**Natural Numbers**

These are the counting numbers: 1, 2, 3, 4, 5,.....

Net

A plane shape that, when folded, can be made into a three-dimensional or solid shape

Nonagon

A plane shape with nine sides

Nonsingular Matrix

A square matrix which has an inverse. A matrix is nonsingular if and only if its determinant does not equal zero.

O**Object Point**

A point before it has undergone a transformation

Obtuse Angle

An angle of more than 90 degrees but less than 180 degrees

Octagon

A plane shape with eight sides

Odd Number

An integer that is not a multiple of 2. The odd numbers are
 $\{ \dots, -3, -1, 1, 3, 5, \dots \}$

One-to-One Function

A function for which every element of the range of the function corresponds to exactly one element of the domain. One-to-one is often written 1-1.

Operation

A way of combining numbers or elements in a set. The most common kind of operation is a binary operation that combines two numbers to give a single number, e.g., for the binary operation '+', $3 + 2 = 5$.

Ordered Pairs

A pair of numbers in which there is a first number and a second number and the order in which they are presented is important. Ordered pairs are usually presented in curly brackets, e.g., $\{a, b\}$.

P**Parallel Lines**

Lines that never meet, no matter how far they are extended

Parallelogram

A *quadrilateral* formed by two pairs of parallel lines

Percentage

A percentage is a way of expressing a fixed fraction of a hundred,
e.g., $10\% = \frac{10}{100}$.

Perfect Square

Any number that is the square of a *rational number*. For example, 0, 1, 4, 9, 16, 25, etc. are all perfect squares. So are $\frac{1}{25}$ and $\frac{9}{4}$.

Perimeter

The distance around the outside of a shape.

Periodic Function

A function which has a graph that repeats itself identically over and over as it is followed from left to right. Formally, a function f is periodic if there exists a number p such that $f(x + p) = f(x)$ for all x .

Perpendicular Bisector

The line perpendicular to a segment passing through the segment's midpoint.

Note: The perpendicular bisectors of the sides of a triangle are concurrent, intersecting at the circumcentre

Perpendicular Lines

Lines that cross at right angles

Pictograph

A pictorial representation of information, similar to a bar graph, but using appropriate symbols to correspond to the frequencies of different kinds of data

Pie Chart

A representation of information in statistics in a visual form, using a circle that is divided into sectors, so that the areas of the sectors represent the data

Place Value

The value of a digit because of its position in the numeral representation of number

Plan

A drawing or diagram that gives a layout of how something is arranged. A design plan is a view from above

Plane

A flat surface. A *plane shape* is a shape that can be drawn with all its points in one plane

Plane Shape

A shape that can be drawn with all its points in one plane

Point

A position in space. It has no size and can be located by giving its coordinates relative to some fixed point of origin

Polygon

A *plane shape* bounded by only straight lines

Polynomial

An algebraic expression containing two or more terms

Population

A target group from which *samples* are drawn for statistical analysis

Power

Same as *index*

Prime Number

A number that has two and only two factors, i.e., itself and 1. Examples of prime numbers are 2, 3, 5, 7, 11, 13, ...

Principal

The sum of money borrowed or invested

Prism

A solid shape with flat faces that are all polygons and has the same cross-sectional shape along its length

Probability

The measure of how likely an event is. The probability of an event is represented by a number between 0 and 1

Product

The result of multiplying two or more numbers

Profit

The difference in value when an item is sold for more than what was paid for it.

Proportion

Two sets are proportional to one another when one set can be written as a *constant* times the other.

Protractor

An instrument used for measuring angles

Pythagoras' Theorem

This states that in a right-angled triangle, the area of the square on the longest side is equal to the sum of the areas of the squares on the two smaller sides.

Q

Quadrant

Each of the four regions into which the x-axis and the y-axis divide the Cartesian plane

Quadratic Equation

An equation includes only second degree polynomials. Some examples are $y = 3x^2 - 5x^2 + 1$, $x^2 + 5xy + y^2 = 1$, and $1.6a^2 + 5.9a - 3.14 = 0$.

Quadratic Expression

A polynomial in which the highest power of the variable(s) is two.

Quadrilateral

A plane shape with four sides

Quartiles

The collective term for the first quartile and third quartile of a set of data. That is, the 25th and 75th percentiles.

QED

An abbreviated Latin phrase used to indicate the end of a proof, especially if it may not be immediately obvious that the proof is complete. QED stands for the Latin phrase *quod erat demonstrandum*, which means "That which was to be proven."

Quotient

The answer you get when you do a division.

R**Radius**

The distance from the centre of the circle to a point on the circumference.

Random

Numbers or objects that are chosen without bias.

Range

The set of numbers onto which the elements of the first set or domain in a *relation* is mapped

Rate

A percentage of the sum of money borrowed or invested, paid on a periodic basis

Ratio

A comparison of two quantities expressed in like units.

Rational Number

A number that can be written in the form $\frac{a}{b}$, where a and b are integers,

e.g., $\frac{5}{8}$, $\frac{12}{3}$, $\frac{7}{1}$.

Real Number

Any number that corresponds to a point on a number line. The set of real numbers is the union of the set of rational numbers and the set of irrational numbers.

Rectangle

A quadrilateral formed by two pairs of parallel lines meeting at right angles

Rectangular Number

A number with two factors that are not equal or equal to 1 — composite numbers. The dots of composite numbers can be lined up to form rectangles.

Recurring Decimal

A decimal where the pattern of digits after the decimal point keeps repeating itself, e.g., 0.3333... or 0.23823823823...

Reflection

A transformation that transforms a shape in the same way as a mirror

Reflex Angle

An angle of size between 180 and 360 degrees

Relation

A rule that connects the elements of two corresponding sets, such as sets of things, numbers, or people

Rhombus

A quadrilateral with four sides of equal length

Right Angle

An angle that measures 90 degrees. A quarter turn

Right Circular Cone

A right cone with a base that is a circle

Root of an Equation

Any number or value for the variable that satisfies the equation. It is the solution to the equation.

Rotation

A transformation that moves points so that they remain the same distance from a fixed point called the centre of rotation

Row of a Matrix

A horizontal set of numbers in a matrix.

S

Sample

A group of people or objects chosen from a larger group to provide data to make predictions about the larger group

Scalar

Any real number, or any quantity that can be measured using a single real number. Temperature, length, and mass are all scalars. A scalar is said to have magnitude but no direction. A quantity with both direction and magnitude, such as force or velocity, is called a vector.

Scale Factor

The ratio of any two corresponding lengths in two similar geometric figures.

Note: The ratio of areas of two similar figures is the square of the scale factor.

The ratio of volumes of two similar figures is the cube of the scale factor.

Sector

The shape enclosed by two radii and an arc of the circle

Set

A collection of well-defined things

Similar Triangles

Triangles with the same shape but different in size

Simple Interest

The amount of money paid or earned on a sum of money over a given period of time

Simultaneous Equations

Two or more independent equations involving more than one variable for which there is a common set of roots

Slope of a Line

A number which is used to indicate the steepness of a line, as well as indicating whether the line is tilted uphill or downhill. Slope is indicated by the letter **m**.

Solid

Geometric Solid

Solid Geometric Figure

The collective term for all bounded three-dimensional geometric figures. This includes polyhedra, pyramids, prisms, cylinders, cones, spheres, ellipsoids, etc.

Speed

The distance travelled by a body per unit of time, e.g., 50 km/h

Sphere

A three dimensional solid consisting of all points equidistant from a given point. This point is the centre of the sphere. Note: All cross-sections of a sphere are circles.

Statistics

The branch of mathematics concerned with the collection, organization, representation, interpretation, and analysis of data and the making of predictions based on the information gathered.

Straight Angle

An angle of 180 degrees

Subsets

If every element of a set B is also an element of the set A, then B is a subset of A.

Substitution

The process of replacing letters in a formula or expression with numbers

Supplementary Angles

Two or more angles that add up to 180 degrees

Surd

An *irrational* number, e.g., $\sqrt{2}$

Surface Area

The total area of the exterior surface of a solid. Many formulas for the area of a surface are given below.

Symmetry

The way that a shape can be reflected or rotated to fit onto itself

T**Tangent**

A straight line that touches a circle or curve at a single point

Temperature

Degree of heat as shown by a thermometer. It is usually measured in degrees Celsius ($^{\circ}\text{C}$) or in degrees Fahrenheit ($^{\circ}\text{F}$).

Terminating Decimal

A fraction that can be worked out exactly to a fixed number of decimal places (i.e., without recurring)

Tessellation

A way of covering a plane shape using a pattern and without leaving gaps

Tetrahedron**Regular Tetrahedron**

A polyhedron with four triangular faces, or a pyramid with a triangular base

Transformation

A change from one position to another position by a defined movement

Translation

A transformation in which a shape slides without turning. Every point moves the same distance in the same direction

Transversal

A line drawn across two or more parallel lines

Trapezium

A quadrilateral with one pair of opposite sides parallel

Triangle

A plane shape with three straight sides

Triangular Number

A number that can be shown as a triangle of dots, e.g., 3, 6, 10...

Trig Functions

The six functions *sine*, *cosine*, *tangent*, *cosecant*, *secant*, and *cotangent*. These functions can be defined several different ways. These include SOHCAHTOA definitions, circle definitions (below), and unit circle definitions.

U

Union

The union of two sets A and B is the set that contains all elements that are in A, or in B, or in both A and B. The union of sets is shown by using the symbol \cup (e.g., $A \cup B$)

Unitary Method

A method of carrying out a calculation to find the value of a number of items by first finding the cost of one of them

Universal Set

The set of all elements being considered

V**Variables**

Letters or symbols that are used to represent any of a variety of different number.

VAT

Value Added Tax. A charge of 15% on specified goods and services

Vector

A quantity, drawn as an arrow, with both direction and magnitude. For example, force and velocity are vectors. If a quantity has magnitude but not direction, it is called a scalar. Temperature, length, and mass are examples of scalars.

Venn Diagram

A visual display that is used to represent sets and their relationship with each other. A typical diagram will show each set represented by a circle inside the *Universal set*, which is represented by a rectangle.

Vertex

The point where the sides of a plane shape or solid meet

Vertically Opposite Angles

A pair of angles that are opposite to each other, where two lines cross

Volume

A measure of the amount of space that a solid occupies. Also called capacity

W**Whole Numbers**

The set of natural or counting numbers in addition to the number 0, e.g., 0, 1, 2, 3, 4, ..

X

***x*-intercept**

A point at which a graph intersects the **x**-axis. The **x**-intercepts of a function must be real numbers, unlike roots and zeros.

Y

***y*-intercept**

A point at which a graph intersects

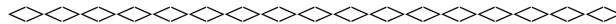
Z

Zero

The identity element under addition of numbers. The first whole number

Zero Matrix

A matrix for which all elements are equal.



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